Mathematical model and algorithm for selection of Internet sites and places for display of communication messages in planning advertising campaigns

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Abstract
This paper discusses the problem of selecting a set of online marketing tools, advertising space, places and duration of display of advertising communications (AC) restricted by a tight advertising budget. The tool offered to evaluate the efficiency of AC display is the conversion rate. The paper reviews the methods for calculating the conversion rate. The formalized problem statement is represented as an integer valued linear programming model and reduced to maximization of the total conversion from advertising communications displayed under a tight advertising budget and with a limited selection of online marketing tools and duration of AC display.

The paper describes a solution algorithm based on the method of wave planning. It allows the decision maker to run an iterative decision making process in each interval of the planning period based on the AC conversion rate obtained at the previous stage. The authors propose methods for calculating input parameters for the model: AC display conversion rate, cost of AC placement in the advertising space over the shortest possible period, average number of AC displays over the shortest possible period.

The paper describes the results of experimental study of the model and the algorithm using the example of AC planning for Electronic Timetable software marketing to colleges and universities of the Kemerovo region.

The results have practical value for executives and marketing managers of small innovation-based companies running advertising campaigns and planning communications with potential consumers using the tools of online marketing.

Key words: online marketing tool, advertising space, advertising communication, display conversion, integer model, wave planning.

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Introduction
Let us assume that a small IT company having a finished software product (SWP) is planning to bring it to market. At the same time, a basic market of potential users is determined, and its target segments are identified. SWP delivery options differentiated by functional and business models for each target market segment are formed, and the structure and composition of advertising communications focused on
consumer preferences of the potential users are developed [1–4]. The company leaders face the task of developing a plan of marketing activities which includes an advertising campaign. Taking into account the high cost of traditional channels for dissemination of advertising (print media, radio, television, telephony and others), it is reasonable to use various online marketing tools, i.e. media advertising, contextual advertising, Internet-PR, participation in partner programs, promotion in social media, search engine optimization, address dispatch of advertisements as a channel of marketing specialists’ communication with the target audience. Under these circumstances, the company management needs to solve the problem of selecting a set of online marketing tools, advertising spaces, display spaces and duration of posting advertising communications within the limited advertising budget.

1. Statement of the problem

In subsequent presentation, the following terminology is used. Advertising space is taken to mean an Internet resource on which an advertising communication can be posted. For spreading advertising communications, it is recommended to simultaneously use several advertising spaces. A specific place designated for display of advertising communications on the advertising space through a specific online marketing tool (display advertising, contextual advertising, Internet-PR and others) is a display position. According to recommendations [5, 6], the communication process should use several online marketing tools, for each of which a certain number of display positions can be available. The advertising campaign is carried out in a specified planning interval defining the duration of the communication impact on representatives of the target audience (number of days, weeks, months, quarters). For each display position, a minimum allowed interval of posting advertising communications is established. The duration of posting advertising communications for each display position is measured in units of the planning interval and is determined as the sum of minimum allowed posting intervals. Taking into account the effect of advertising “burn-out”, it is necessary to limit the duration of posting the advertising communications for each display position [7].

Target action is an expected response of the target audience representatives to reading advertising communications: go to a website that contains information on SWP, download a SWP demo, execute an online-order for purchasing a SWP, etc. The effectiveness of displaying advertising communications in each position is assessed using the conversion rate, which may be calculated according to the following options [6]:

- relation of the number of users who received advertising communications to the total number of Internet users;
- relation of the number of users who paid attention to the advertising communication to the number of users to whom it was delivered;
- relation of the number of interested users to the number of users who paid attention to the advertising communication;
- relation of the number of users who visited the SWP Internet resource to the number of interested users;
- relation of the number of users who performed a target action on the SWP Internet resource to the number of visitors to the Internet resource;
- relation of the number of users who revisited the Internet resource to the number of visitors.

In this case, in accordance with the stages of user interaction with the advertising communication (demonstration of advertising communication, highlighting, motivation, visiting the Website, action, repetition) listed in the literature [8], the conversion rate is calculated as the relation of the number of representatives of the target audience who performed a targeted action to the number of users who have contact with the advertising communication expressed as a percentage.

Based on the definitions introduced, the problem of selecting positions and duration of posting the advertising communications can be formulated in the form of the following mathematical model. Let us assume that assigned are: \( R \) – advertising budget; \( T \) – duration of the planning interval; \( I = \{1, \ldots, n\} \) – a set of advertising spaces; \( J = \{1, \ldots, m\} \) – a set of tools disseminating advertising communications; \( L = \{l_j\}, j = 1, n, j = 1, m \) – a set of available display positions in the \( i \)-th advertising space through the \( j \)-th tool.

In this case, each display position is characterized as follows:

- \( t_{ip} \) – minimum allowed interval of posting the advertising communication in the \( p \)-th position;
- \( T_{ip} \) – maximum duration of posting the advertising communication in the \( p \)-th position;
- \( c_{ip} \) – advertising cost for a minimum allowed interval in the \( p \)-th position;
- \( v_{ip} \) – average number of displays of the advertising communication for a minimum allowed interval of placement in the \( p \)-th position;
- \( k_{ip} \) – display conversion in the \( p \)-th position.
It is necessary to determine a set 
\( X = \{ x_{ijp} \} \), \( i = 1, n, j = 1, m, p = 1, l_{ij} \) while maximizing 
the total conversion through displays of the advertising 
communications (1) and implementing a set of con-
straints (2–5), where \( x_{ijp} = \{ 0, 1, 2, \ldots, d \} \) is a number 
of minimum allowed intervals of posting the advertising 
communication in the \( p \)-th position of the \( j \)-th advertising 
space using the \( j \)-th tool in maximum achievement 
of a number of desired actions:

\[
Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{p=1}^{l_{ij}} k_{ijp} \cdot x_{ijp} \cdot v_{ijp} \rightarrow \max. \quad (1)
\]

The total cost of posting the advertising communications 
should not exceed the advertising budget:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{p=1}^{l_{ij}} c_{ijp} \cdot x_{ijp} \leq R. \quad (2)
\]

To disseminate advertising communications, all on-
line marketing tools selected by decision-makers should 
be used:

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} l_{ijp} \geq 1, j = 1, m. \quad (3)
\]

The duration of posting advertising communications 
on each advertising space using a specific tool should not 
exceed the duration of the planning interval:

\[
\sum_{p=1}^{l_{ijp}} x_{ijp} \cdot t_{ijp} \leq T, i = 1, n, j = 1, m. \quad (4)
\]

The duration of posting advertising communications 
in each display position should not exceed the threshold 
set by the decision-maker:

\[
x_{ijp} \cdot t_{ijp} \leq T_{ijp}, \quad i = 1, n, j = 1, m, p = 1, l_{ij}. \quad (5)
\]

The mathematical model presented (1–5) is an inte-
ger linear programming problem and can be solved us-
ing a table processor that is part of an office suite of ap-
lications (Microsoft office Excel, OpenOffice Calc and 
others) or specialized mathematical packages, including 
Linear Program Solver (LiPS) and MATLAB.

2. Problem solution algorithm

The algorithm is based on the wave planning proce-
dure [5, 9]. This selection is prompted by the lack of 
empirical data on the quality of communication ac-
tions at the stage of introducing a product to the market 
and possible inaccuracies in determining the average 
number of displays. The procedure consists in divid-
ing the duration of the advertising campaign into short 
intervals (waves) and planning subsequent next wave 
after implementation of the previous one. This allows 
the company management to analyze the results ob-
tained at the previous step, and to adjust the conver-
sion projections for the next wave. In consideration of 
the foregoing, the algorithm for solving the problem of 
selecting advertising spaces and positions of advertising 
communications for each target segment can be pre-
sented as a sequence of steps.

Step 1. Determining the characteristics of the current 
wave. The decision-maker determines the current wave 
planning interval and specifies the duration of the plan-
ing interval, the amount of the advertising budget and 
the list of online marketing tools used for disseminating 
the advertising communications.

Step 2. Determining a list of display positions. The 
decision-maker selects a list of advertising spaces and 
possible display positions for each of them. The values 
of cost parameters for posting an advertising commu-
nication, minimum allowed placing interval and average 
amount of displays for an allowed posting interval are 
determined for each display position.

Step 3. Determining projected values of placement rates 
for advertising communications. The decision-maker 
specifies a projected display conversion and the maxi-
mum duration of placement for each display position.

Step 4. Selecting positions and duration of displays. 
The problem solution (5.1) using one of the software 
packages. The result obtained is taken to be the optimal 
plan of placing the advertising communications in the 
current planning interval.

Step 5. Implementing the posting plan. In accordance 
with the result obtained in Step 4, activities on posting 
the advertising communications are performed. During 
the advertising campaign, the decision-maker collects 
the achieved indicators and analyzes their deviations 
from the values predicted in Step 3.

Step 6. Planning the next wave. After implementing the 
current wave of the advertising communications plan, a 
transition to Step 1 with data correction in Steps 2 and 3 
is performed, or a decision is taken on completion of the 
advertising campaign.

3. Calculation methods 
for initial model parameters

To predict the conversion rate, the Project Evaluation 
and Review Technique (PERT) will be used [10]. The 
essence of this technique is that in determining the pro-
jected value of the conversion rate the decision-maker gives three estimations: \( o \) — optimistic; \( p \) — pessimistic; \( b \) — realistic. The optimistic estimation of the conversion assumes that the advertising communication is implemented and posted in such a way that the target audience representatives will pay attention to it, be interested in the product and, with a high degree of probability, will perform a desired action. The pessimistic estimation of the conversion is made under the assumption that most of the receivers of the advertising communication will demonstrate little interest, and only a small part of them will perform a desired action. A realistic estimation of the conversion means the most probable estimation performed based on statistically average values of the conversions representative of the Internet marketing tools used. The average estimation of conversion prediction is determined by multiplying the realistic estimation by 4, adding the optimistic and pessimistic estimations and dividing the result by 6.

The cost of placing advertising communications is determined based on the existing cost models of advertising on the Internet. As of today, four basic cost models can be identified: a fixed cost for advertising for a certain period (Flat Fee Advertising, FFA), cost for a thousand displays of the advertising communication (Cost Per Thousand, CPM, where \( M \) is a Roman designation of a thousand), Cost per Click (CPC) and Cost per desired Action (CPA) [11].

In this case, the FFA model is used as a reference model for determining the cost parameter of posting advertising communications for the minimum allowed interval. Recalculation of the cost of advertising based on FFA models depending on other established models of advertising spaces is determined by the formulas (6–8).

For \( C_{PM} \) model the formula is as follows

\[
C_{\text{FFA}} = \frac{V \cdot C_{\text{CPM}}}{1000},
\]

where \( V \) — average number of displays of advertising communications for a minimum allowed posting interval;

\( C_{\text{CPM}} \) — fixed cost of a thousand displays.

For the \( C_{PC} \) model, the formula is as follows

\[
C_{\text{FFA}} = V \cdot C_{\text{CPC}} \cdot CTR,
\]

where \( C_{\text{CPC}} \) — cost of one click;

\( CTR \) — conversion of transitions (“clicks”) relative to displays of advertising communication.

For the \( C_{\text{PA}} \) model, the formula is as follows

\[
C_{\text{FFA}} = V \cdot C_{\text{CPA}} \cdot CTA,
\]

where \( C_{\text{CPA}} \) — cost of a desired action;

\( CTA \) — conversion of performing desired actions to displays of advertising communication.

Data on the average number of displays for a certain period of time is generally published at advertising spaces, and can also be obtained from the owners of the advertising space or from statistical data of external systems of web analytics, such as Google Analytics, Yandex.Metrics, Liveinternet and others.

4. Experimental studies

The experimental studies of the proposed models and algorithms were conducted on the example of developing an advertising communications plan while promoting SWP “Electronic training agenda” in secondary schools and higher educational institutions of the Kemerovo region. To post advertising communications at advertising spaces of the news portal “GOROD NOVOSTEY. News Agency” (City-N.ru), portal “NCW (Independent City Website) Kemerovo” (ngs42.ru), Kemerovo’s Newspaper (a42.ru), news portal “Electronic Kuzbass” (e-kuzbass.ru), the information-entertainment portal “KHUTOR” (hutor.ru) and the social network “Vkontakte” were selected. Media advertising and Internet-PR should be used as tools for advertising communications. The dimension of the problem of these initial data was 38 variables and 41 restrictions.

The following parameters have been identified for the first wave of the advertising plan:

- duration of posting advertising communications — 7 days;
- advertising budget — 17,100 rubles;
- online marketing tools — Internet-PR and media advertising;
- allowable display period — from 1 to 30 days;
- maximum duration of displays — from 3 to 7 days;
- projected conversion — from 0.001 to 0.02 %;
- cost of posting FFA over the allowable planning interval — from 250 to 21,000 rubles.

As a result of solving the mathematical problem by means of a table processor of the office software suite LibreOffice Calc, the following decision was taken on arranging advertising communications for SWP promotion in the higher educational institutions in the Kemerovo Region: posting advertorials on news portals “GOROD NOVOSTEY” (city-n.ru) and “Electronic
In experimental studies, the model was tested only during the planning of one advertising wave. For this reason, the impacts of changes in the decision-maker’s projected conversion on the solution results in the next waves were not studied. With the expansion of practical use of the model, the study of this conformity may be of interest.

**Conclusion**

The mathematical model and algorithm presented for selecting positions for advertising communications and duration on Internet sites make it possible to increase the number of desired actions of potential SWP customers using a set of online marketing tools within a limited budget. On the practical level, use of the study results allows managers of small innovative enterprises to increase the effectiveness of advertising campaigns through sequential optimization of the projected conversion rates for desired actions of the target audience representatives.

**References**

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Аннотация
В статье рассмотрена задача выбора комплекса инструментов Интернет-маркетинга, рекламных площадок, мест показа и продолжительности размещения на них коммуникационных сообщений в условиях ограниченного рекламного бюджета. Результативность показов на каждом месте предлагается оценивать с помощью показателя конверсии. Приведен обзор вариантов расчета показателя конверсии. Формализованная постановка задачи представлена в виде целочисленной модели линейного программирования и сведена к максимизации суммарной конверсии от показа коммуникационных сообщений при ограничениях на размер рекламного бюджета, количество используемых инструментов Интернет-маркетинга, продолжительность их размещения на рекламных площадках и местах показа.

Описана алгоритм решения задачи, в основу которого положена методика волнового планирования. Это позволяет лицу, принимающему решение (ЛПР), организовать итерационный процесс поиска решения в каждом интервале планируемого периода с учетом полученной на предыдущем шаге конверсии показа. Предложены методы расчета исходных параметров модели: конверсий показов, стоимости размещения коммуникационных сообщений на рекламной площадке за минимально допустимый интервал размещения, среднего количества показов за минимально допустимый интервал.

Описаны результаты экспериментальных исследований модели и алгоритма на примере разработки плана размещения коммуникационных сообщений при продвижении программного продукта (ПП) «Электронное расписание занятий» в ссузах и вузах Кемеровской области. В практическом плане полученные результаты могут быть полезны руководителям и маркетологам малых инновационных предприятий при проведении рекламных кампаний и планировании мероприятий по коммуникационному воздействию на потенциальных потребителей с использованием комплекса инструментов Интернет-маркетинга.

Ключевые слова: инструмент Интернет-маркетинга, рекламная площадка, коммуникационное сообщение, конверсия показа, целочисленная модель, волновое планирование.


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