Research into the dynamics of railway track capacities in a model for organizing cargo transportation between two node stations

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Abstract

The article deals with a model for organizing railway transportation on a long stretch of road between two node stations connected by a large number of intermediate stations. Between two arbitrary neighboring stations, there is a railway track for temporary storage of cargo. The movement of cargo is carried out in one direction. To ensure the smooth movement of cargo, two technologies are used which are common for all stations. The first technology is based on the procedure of interaction of a station with both neighboring stations and adjacent railway tracks. The second technology uses the technical capabilities of the station itself and is based on the interaction of the station with neighboring railway tracks. For cargo transportation, a simple control system is used which provides for measuring the volume of transported goods at neighboring stations with a single time lag.

This work is devoted to describing and studying the dynamics of the number of roads involved in the railway tracks. For this purpose, a system of differential equations is formed, the right parts of which are functions of variables describing the dynamics of the number of roads involved in the stations. The starting point for this study is previously obtained results from studying the dynamics of the number of tracks involved in the stations (a brief description of these results is given in the Introduction). What follows is the description of the dynamics of the number of roads involved in the railway tracks.

Possible variants of the dynamics (growth of the number of the roads involved on one railway tracks and falling on others) and their dependence on parameters of the model are investigated. We also study the dependence of the rate of change in the number of involved roads on the railway tracks on the model parameters. We then find the parameter of control by which it is possible to provide arbitrarily small speed of growth (fall) of the number of the roads involved on all railway tracks.

Key words: station; railway track; organization of cargo transportation; mathematical model; differential equations; dynamics; numerical realization.

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Introduction

Transport is one of the main branches of any state and performs the connecting, communication and supply functions. For the correct organization of traffic in the transport network, control systems are used. Their algorithms are based on mathematical models, one of the main functions of which is the modeling of traffic flows. A large number of publications are devoted to mathematical modeling of traffic flows. Works [1–3] describe “analog models” in which the movement of the vehicle is similar to any physical flow (hydro- and gas-dynamic models). There are a large number of models designed to optimize the functioning of transport networks [4–7]. This class of models solves the problems of optimization of transportation routes, development of optimal configuration of the transport network, etc. One of the approaches to modeling and research of traffic flows is based on the theory of competitive non-coalition equilibrium [8–11]. It allows us to describe a fairly adequate mechanism for the functioning of road networks. We also note the approach associated with the use of simulation and cellular automata described in [12–15]. Recently, an alternative theory of transport flows has been actively developed, called the theory of three phases (classical theories consider two phases: free flow and dense flow) [16–20]. This theory can predict and explain the empirical properties of the transition to dense flow and the resulting space-time structures in the transport flow.

A number of publications are devoted to the modeling of rail traffic and related transport flows [21–27]. In particular, in works [24–27] a model of organization of rail freight between two node stations connected by a railway line which contains a certain number of intermediate stations is investigated. It is assumed that between stationary stations there is interexchange railway track, where part of the cargo can be temporarily stored (in a special storage area). The movement of goods is carried out in one direction. The traffic flow diagram is shown in *Figure 1.*

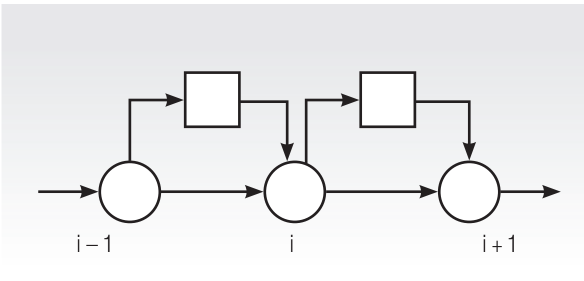

Fig. 1. Scheme of freight traffic of railway transport

In this figure, the circles indicate the stations, and the squares indicate the railway tracks. As can be seen from the figure, cargo can arrive at an arbitrary intermediate station both from the previous station and from the railway track, after which the cargo can be sent either to the next station or to the railway track. Let the number of intermediate stations be equal to m. Denoting by 0 and $m + 1$ respectively the numbers of the initial and final nodal stations, we obtain the following set of station numbers: $\{0, 1, ..., m, m + 1\}$. Each station at any time is characterized by the number of involved roads. Denote by $z_i(t)$, $i = 0, 1, ..., m + 1$ number of roads involved in the i-th station at time t. The maximum number of involved roads at the stations, at which the mode of increasing the number of roads at the expense of goods from the railway track, is functioning, we denote by Δ. If the number of paths involved exceeds the maximum value, then part of the cargo is temporarily sent to the storage area.

The organization of cargo traffic is carried out using two technologies.

The first technology is based on the interaction procedure of neighboring stations. The following rule applies here: an arbitrary station can send cargo to the next station if the number of involved roads is greater than at the next station. In this case, the intensity of shipment is proportional to the difference in the number of involved roads at these stations. Note that sending goods from an arbitrary station (except the last) with a certain intensity is equivalent to receiving goods with the same intensity at the next station. Thus, each station with a number i $(1 \le i \le m)$ can take the cargo from the previous station with an intensity equal to $\alpha(z_{i-1} - z_i)$, if $z_{i-1} > z_i$ and send the cargo to the next station with an intensity equal to $\alpha(z_i - z_{i+1})$, if $z_i > z_{i+1}$. If the first condition is violated, the station with number i sends the cargo to the railway track with intensity $\alpha(z_i - z_{i-1})$, and if the second one is violated, it receives the cargo from the railway track with intensity $\alpha(z_{i+1} - z_i)$. Initial node station ($i = 0$) takes a cargo with intensity $\psi_1(t)$ and sends it to the next station with intensity $\alpha(z_0 - z_1)$ if $z_0 > z_1$. Otherwise, the initial node station additionally takes the cargo with the intensity $\alpha(z_1 - z_0)$. Final node station ($i = m + 1$) accepts the load from the previous station with intensity $\alpha(z_m - z_{m+1})$, if $z_m > z_{m+1}$, and distributes it with intensity $\psi_2(t)$. If $z_m < z_{m+1}$, then the final station additionally distributes the cargo with intensity $\alpha(z_{m+1} - z_m)$.

The second technology is designed to use the infrastructure capabilities of the stations and to ensure uninterrupted movement of cargo. It is based on the procedure of interaction of the station with neighboring railway track located on opposite sides of it. The second technology for all stations, except the initial one, allows us to increase the number of involved roads (if it does not exceed Δ), and to reduce it (if it exceeds Δ). The function $\varphi(.)$, setting the speed of change of number of involved roads within this technology has the following properties: on a half-line $(-\infty, 0]$ it is identically equal to zero, on an interval $(0, x_{opt})$ is increasing, in a point x_{opt} accepts the maximum value, on a half-line $(x_{opt}, +\infty)$ is decreasing, in a point Δ accepts zero value, and on a half-line $(\Delta, +\infty)$ is linear. For the initial node station ($i = 0$) the second technology is used only for unloading. The function $\varphi_0(.)$, setting the speed of change of the number of involved roads at this station within this technology, has the following properties: on a half-line $(-\infty, \Delta]$ it is identically equal to zero, and on a half-line $(\Delta, +\infty)$ it is linearly decreasing.

For cargo transportation, a simple control system is used: the quantity of involved roads at any station has to coincide with the quantity of involved roads at the following station, with a time log which is uniform for all stations.

Thus, the dynamics of numbers of the involved roads at stations is set by the system of the differential equations

$$\dot{z}_0(t) = \psi_1(t) - \alpha z_0 + \alpha z_1 + \varphi_0(z_0),$$
$$t \in [0, +\infty), \quad (1)$$

$$\dot{z}_i(t) = \alpha z_{i-1} - 2\alpha z_i + \alpha z_{i+1} + \varphi(z_i),$$
$$i = 1, 2, ..., m, \quad t \in [0, +\infty), \quad (2)$$

$$\dot{z}_{m+1}(t) = \alpha z_m - \alpha z_{m+1} - \psi_2(t) + \varphi(z_{m+1}),$$
$$t \in [0, +\infty), \quad (3)$$

and a control system — by nonlocal linear restrictions

$$z_i(t) = z_{i+1}(t+\tau), \quad i = 0, 1, 2, ..., m,$$
$$t \in [0, +\infty). \quad (4)$$

The constant τ will be called a characteristic of the control system.

Definition 1. The family of absolutely continuous functions $\{z_i(.)\}_0^{m+1}$, defined on $[0, +\infty)$, is called the solution of the traveling wave type with characteristic τ (soliton solution), if almost all $t \in [0, +\infty)$ functions $z_i(.)$ satisfy the system (1) — (3) and nonlocal restrictions (4).

The class of soliton solutions is extremely narrow. This leads to the need to properly extend the class of soliton solutions to the class of soliton quasi-solutions. In [24—27] two ways of such expansion are proposed. One type of expansion involves the assumption of discontinuous soliton solutions (we call them soliton quasi-solutions of the first type).

Definition 2. The family of absolutely continuous functions $\{z_i(.)\}_0^{m+1}$, defined on $[0, +\infty)$; it is called a soliton quasi-solution of the first type with characteristic τ, if almost all $t \in [0, +\infty)$ functions $z_i(.)$ satisfy the system (1) — (3) and nonlocal restrictions (4), with possible discontinuities at the points $k\tau \;\; k = 1, 2, ...$.

It is proved $\overline{\tau} > 0$, that for any $\tau \in (0, \overline{\tau})$, $\overline{i} \in \{0, 1, ... m, m+1\}$ system (1) — (4) with a fixed initial value $z_{\overline{i}}(\overline{t})$ at the initial time $\overline{t}$ has a single "quasi-solution" of the first type [24].

Definition 3. A soliton quasi-solution of the first type with a characteristic τ is called ε-quasi-solution of the first type with characteristic τ if inequalities

$$| z_0(k\tau - 0) - z_0(k\tau + 0) | < \varepsilon,$$

are satisfied, for all $k = 1, 2, ...$.

It is proved $\overline{\tau} > 0$ that for any $\tau \in (0, \overline{\tau})$ there is ε-soliton quasi-solution of the first type with a characteristic τ with however small $\varepsilon > 0$ [24].

The second type of expansion of soliton solutions allows weakening of the control system (implementation of nonlocal restrictions (4) with some error). We give an exact formulation of quasi-solutions of this type.

Definition 4. The family of absolutely continuous functions $\{z_i(.)\}_0^{m+1}$, defined on $[0, +\infty)$, is called ε-soliton quasi-solution of the second type with characteristic τ, if almost all $t \in [0, +\infty)$ functions $z_i(.)$ satisfy the system (1) — (3) and the condition

$$| z_i(t) - z_{i+1}(t+\tau) | < \varepsilon$$
$$i = 0, 1, 2, ..., m, \quad t \in [0, +\infty).$$

is satisfied.

It is proved that the solutions of the system of differential equations (1) — (3) are limited under the limitation of functions $\psi_1(.)$ end $\psi_2(.)$ [27].

In work [27] by means of computer realization quasi-solutions of the second type were investigated for periodic functions

$$\psi_1(t) = \psi_2(t) = d + \gamma \cos(\omega t), \; d \geq \gamma > 0,$$

and also functions $\varphi(.)$ и $\varphi_0(.)$, defined as follows:

$$\varphi(x) = \begin{cases} 0 \;\text{ if }\; x < 0 \\ ax(\Delta - x), \; a > 0 \;\text{ if }\; x \in [0, \Delta] \\ -c(x - \Delta), \; c > 0 \;\text{ if }\; x \in (\Delta, +\infty), \end{cases}$$

$$\varphi_0(x) = \begin{cases} 0 \;\text{ if }\; x \leq \Delta \\ -c_0(x - \Delta), \; c_0 > 0 \;\text{ if }\; x \in (\Delta, +\infty). \end{cases}$$

For this purpose, the set of all solutions of the system of differential equations (1) — (3) was investigated. According to the results of numerical experiments, starting from a certain point in time $\overline{t} > 0$ the solutions of the system (1) — (3) begin to oscillate in some neighborhood of the value Δ, and the components of the solution satisfy the condition $z_0(t) > z_1(t) > ... > z_m(t) > z_{m+1}(t)$ for any $t \in [\bar{t}, +\infty)$.

Moreover, there is an integer $0 \le \bar{m} < m + 1$

$$z_i(t) > \Delta, \text{ if } 0 \le i \le \bar{m}, t \in [\bar{t}, +\infty), \quad (5)$$

$$z_i(t) < \Delta, \text{ if } \bar{m} < i \le m + 1, t \in [\bar{t}, +\infty). \quad (6)$$

Numerical experiments showed that the value $\bar{m}$ depends on parameters c_0, c и a, but does not depend on parameter α. Dependence on parameters c_0 and c is non-increasing: with increasing parameter c_0 value $\bar{m}$ decreases to $\bar{m} = 0$, and with increasing parameter c to $\bar{m} = 1$. Dependence on parameters a is non-decreasing: with its increase $\bar{m}$ increases to $\bar{m} = m$.

The dependence of the solutions of the system of differential equations (1) – (3) on the parameter α is studied. It is shown that for an arbitrary characteristic $\tau > 0$, increasing the parameter α, it is possible to make an arbitrarily small error in the performance of nonlocal restrictions (4).

In the research carried out, it was supposed that capacities of railway tracks (number of the involved roads on them) are unlimited as a result of which observation of their dynamics was not made. However, in fact this assumption is unrealistic: at least, during a long period of time capacities of railway track have to be limited reasonably. This work is devoted to research into the dynamics of capacities of railway tracks and its dependence on model parameters.

1. Description of the dynamics of the railway tracks' capacities

We investigated the dynamics of capacities of railway tracks within the model described in the Introduction. Let's begin with their numbering. The railway track located between stations with numbers i and $i+1$ we will designate number i. Thus, we get the following set of railway tracks numbers: $\{0, 1, ..., m\}$. The number of the involved ways on i-th railway track at the moment of time t we will designate through $y_i(t)$. Determine with what intensity cargo come on the railway tracks and with what intensity leave them. Note that the cargo can be delivered to the railway tracks and sent from them in the framework of both the first and the second technology.

Within the first technology, on a stage with number i $(1 \le i \le m - 1)$ cargo arrives from the station with number i with intensity $\alpha(z_i - z_{i-1})$, if $z_i > z_{i-1}$, and goes to the station with number $i + 1$ with intensity $\alpha(z_{i+2} - z_{i+1})$, if $z_{i+2} > z_{i+1}$. On an initial railway track $(i = 0)$ within the first technology cargo does not arrive. At last, on a final railway track $(i = m)$ within the same technology cargo arrives from the station with number $i = m$ with intensity α $(z_m - z_{m-1})$, if $z_m > z_{m-1}$. The cargo is not sent from the final railway track within this technology.

Within the second technology, on a railway track with number i $(1 \le i \le m - 1)$ cargo arrives from the station with number i with intensity $-\varphi(z_i)$, if the number of the involved ways at the station with number i exceeds value Δ, and goes to the station with number $i+1$ with intensity $\varphi(z_{i+1})$, if the number of the involved ways at the station with number $i+1$ is less than value Δ (the station with number $i+1$ accepts cargo from a railway track). On an initial railway track $(i = 0)$ within the second technology cargo arrives from the initial node station with intensity $-\varphi_0(z_0)$, if the number of the involved roads at the specified station exceeds Δ, and goes to the station with number $i = 1$ with intensity $\varphi(z_1)$, if the number of the involved roads at the station with number $i = 1$ is less Δ. At last, on a final railway track $i = m$ within the second technology cargo arrives with intensity $-\varphi(z_m)$ from the station with number $i = m$ and goes to the final node station $(i = m + 1)$ with intensity $\varphi(z_{m+1})$, if the number of the involved roads at the final node station is less Δ.

Thus, the dynamics of the number of the involved roads on a railway track is described by the following system of differential equations:

$$\dot{y}_0(t) = -\alpha(z_2 - z_1)\operatorname{sign}(z_2 - z_1) -$$
$$\varphi(z_0)\operatorname{sign}(z_0 - \Delta) - \varphi(z_1)\operatorname{sign}(\Delta - z_1),$$
$$t \in [0, +\infty), \quad (7)$$

$$\dot{y}_i(t) = \alpha(z_i - z_{i-1})\operatorname{sign}(z_i - z_{i-1}) -$$
$$-\alpha(z_{i+2} - z_{i+1})\operatorname{sign}(z_{i+2} - z_{i+1})$$
$$-\varphi(z_i)\operatorname{sign}(z_i - \Delta) - \varphi(z_{i+1})\operatorname{sign}(\Delta - z_{i+1}),$$
$$i = 1, 2, \ldots m-1, \; t \in [0, +\infty), \quad (8)$$

$$\dot{y}_m(t) = \alpha(z_m - z_{m-1})\operatorname{sign}(z_m - z_{m-1}) -$$
$$-\varphi(z_m)\operatorname{sign}(z_m - \Delta) - \varphi(z_{m+1})\operatorname{sign}(\Delta - z_{m+1}),$$
$$t \in [0, +\infty), \quad (9)$$

$$\text{where } \operatorname{sign}(t) = \begin{cases} 1, & t > 0 \\ 0, & t \le 0. \end{cases}$$

We investigated a system (7) — (9) on the assumption that components of quasi-solutions of the second type participate in the right parts of the equations.

Using inequalities (5) — (6) and definitions of functions $\varphi(.)$ and $\varphi_0(.)$, we will transform the equations. In particular, from inequalities (5) — (6) it follows that, since the moment $\bar{t}$ all composed a look $\alpha(z_{k+1} - z_k)\operatorname{sign}(z_{k+1} - z_k)$ in the right part of the equations (7) — (9) will be equal to zero. Depending on value $\bar{m}$ we will consider several cases.

The first case: $\bar{m} = 0$. It means that $z_0(t) > \Delta$, $z_i(t) < \Delta$, $i = 1, \ldots, m+1$ for all $t \ge \bar{t}$, and the equation (7) — (9) take a form:

$$\dot{y}_0(t) = c_0(z_0 - \Delta) - az_1(\Delta - z_1),$$
$$z_0 > \Delta, \; z_1 < \Delta, \quad t \in [\bar{t}, +\infty), \quad (10)$$

$$\dot{y}_i(t) = -az_{i+1}(\Delta - z_{i+1}),$$
$$z_{i+1} < \Delta, \; i = 1 \ldots, m, \; t \in [\bar{t}, +\infty). \quad (11)$$

The second case: $1 < \bar{m} < m$

$$\dot{y}_0(t) = c_0(z_0 - \Delta), \; z_0 > \Delta, \; t \in [\bar{t}, +\infty), \quad (12)$$

$$\dot{y}_i(t) = c(z_i - \Delta), \; z_i > \Delta,$$
$$i = 1, \ldots, \bar{m}-1, \; t \in [\bar{t}, +\infty), \quad (13)$$

$$\dot{y}_{\bar{m}}(t) = c(z_{\bar{m}} - \Delta) - az_{\bar{m}+1}(\Delta - z_{\bar{m}+1}),$$
$$z_{\bar{m}} > \Delta, \; z_{\bar{m}+1} < \Delta, \; t \in [\bar{t}, +\infty), \quad (14)$$

$$\dot{y}_i(t) = -az_{i+1}(\Delta - z_{i+1}),$$
$$z_{i+1} < \Delta, \; i = \bar{m}+1 \ldots, m, \; t \in [\bar{t}, +\infty). \quad (15)$$

The third case: $\bar{m} = m$

$$\dot{y}_0(t) = c_0(z_0 - \Delta), \; z_0 > \Delta, \; t \in [\bar{t}, +\infty), \quad (16)$$

$$\dot{y}_i(t) = c(z_i - \Delta), \; z_i > \Delta,$$
$$i = 1, \ldots, \bar{m}-1, \; t \in [\bar{t}, +\infty), \quad (17)$$

$$\dot{y}_m(t) = c(z_m - \Delta) - az_{m+1}(\Delta - z_{m+1}),$$
$$z_m > \Delta, \; z_{m+1} < \Delta, \quad t \in [\bar{t}, +\infty). \quad (18)$$

Directly from (10) — (18) it follows that in all three cases the right parts of all equations (except for, perhaps, equation with number $\bar{m}$) either are positive, or are negative. Numerical experiments showed that in the first a case ($\bar{m} = 0$) the right member of equation with number $\bar{m} = 0$ is positive. It is connected with the fact that this case takes place if the value of parameter c_0 is significantly more than the value of parameter a. In the third case $\bar{m} = m$) the right member of equation with number $\bar{m}$ is negative. It is connected with the fact that this case takes place if the value of parameter c is significantly less than value of parameter a.

Thus, in the first case the right parts of equation with number $\bar{m} = 0$ are positive, and the right parts of other equations are negative. Therefore, in this case the number of the involved roads on a railway track with number $\bar{m} = 0$ will increase indefinitely, and the number of the involved roads on other railway track will decrease indefinitely.

In the third case, the right part of the equation with the number $\overline{m} = m$ is negative, and the right parts of the remaining equations are positive. Accordingly, in this case, the number of involved roads on the railway track with the number $\overline{m} = m$ will decrease indefinitely, and the number of involved roads on the remaining railway tracks will increase indefinitely.

In the second case, the right parts of the equations with numbers less than $\overline{m}$ are positive, the right parts of the equations with numbers more than $\overline{m}$ are negative. The right part of the equation with the number $\overline{m}$ can be both positive and negative, and with certain combinations of parameters can be equal to zero. Therefore, in this case only on one railway track the number of the involved roads cannot change over time. The number of involved roads on the remaining railway tracks will either increase indefinitely or decrease indefinitely. For example, *Figure 2* shows the dynamics of the number of involved roads in the railway tracks in case of constant functions describing the intensity of the supply of cargo to the initial node station and the intensity of the distribution of cargo from the final node station, i.e. $\psi_1(t) = \psi_2(t) = d$, $d > 0$ (case 2, equations (12) – (15)). The number of stations is equal to 10, respectively, the number of railway tracks is 9 ($y_0, y_1, ..., y_8$ – the number of involved roads on these railway tracks). The value Δ that determines the capacity of the stations is equal to 10, and the parameters have the following values: $\alpha = 10$, $a = 0.1$, $c_0 = c = 1$, $d = 3$. Numerous experiments have shown that all conclusions regarding the dynamics of the capacity of the railway tracks, which will be given below, are valid for any other number of stations (railway tracks) and values Δ.

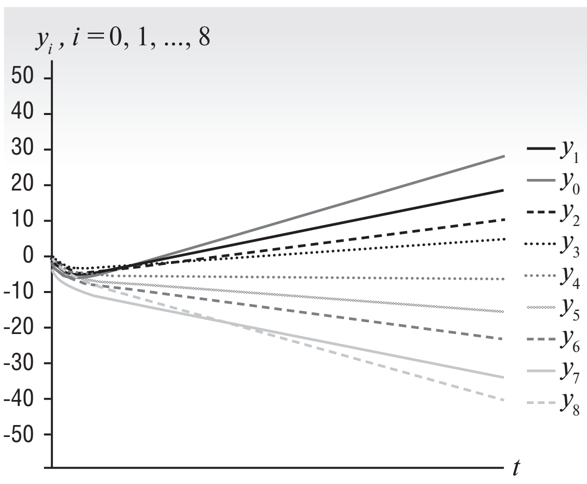

Fig. 2. Dynamics of the number of involved roads on the railway tracks with constant functions $\psi_1(t)$ and $\psi_2(t)$

For periodic functions $\psi_1(t) = \psi_2(t) = d + \gamma \cos(\omega t)$, $d \geq \gamma > 0$ the dynamics of the number of involved roads on the railway tracks does not change fundamentally. For example, *Figure 3* shows the dynamics for the following parameter values: $\alpha = 10$, $a = 0.1$, $c_0 = c = 1$, $d = 3$, $\gamma = 3$.

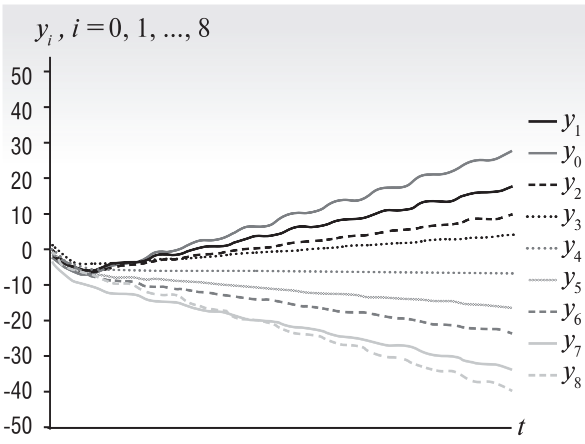

Fig. 3. Dynamics of the number of involved roads on the railway tracks with periodic functions $\psi_1(.)$ and $\psi_2(.)$

In this regard, further research will be carried out for the case of constant and equal functions $\psi_1(t)$ and $\psi_2(t)$.

2. Dependence of growth rate and falling number of involved roads on the railway tracks of the model parameters

We investigated the dependence of the growth rate and the fall of the number of involved roads on the railway tracks on the model parameters. Let's start with the parameter c_0. Recall that this parameter determines the intensity of the shipment of cargo from the initial node station at the zero railway track. Let $\overline{m} \geq 1$(equations (12) – (18)). As shown by numerical experiments, an increase in this parameter leads to an increase in the rate of growth of the number of involved roads on the zero railway track, a decrease in the rate of growth of the number of involved roads on the railway tracks with numbers 1, ..., $\overline{m}$, and an increase in the rate of fall on the following railway tracks. At the same time, both the decrease in the rate of growth of the number of involved roads on the railway tracks with numbers 1, ..., $\overline{m}$ and the increase in the rate of fall on the following railway tracks weaken with the increase in the number of railway tracks. This trend can be seen in *Figure 4*, where the parameter value c_0 is increased to two, with unchanged values of other parameters (α = 10, a = 0.1, c_0 = 2, c = 1, d = 3).

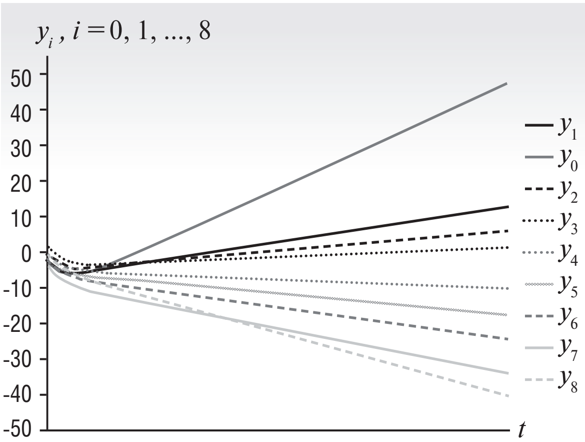

Fig. 4. Dynamics of the number of the involved roads on railway tracks at increase of value of parameter c_0 (double increase)

Let's remember that the value $\overline{m}$ depends on parameter c_0: at its increase the value $\overline{m}$ decreases to $\overline{m}$ = 0. Therefore, in the process of increases to this parameter, growth of the number of involved roads on all stages railway tracks, except for initial, is replaced with falling numbers. This trend can be seen in *Figure 5* (the equations (10) – (11)). In it the value of parameter c_0 is increased up to 60 at invariable values of other parameters (α = 10, a = 0.1, c_0 = 60, c = 1, d = 3).

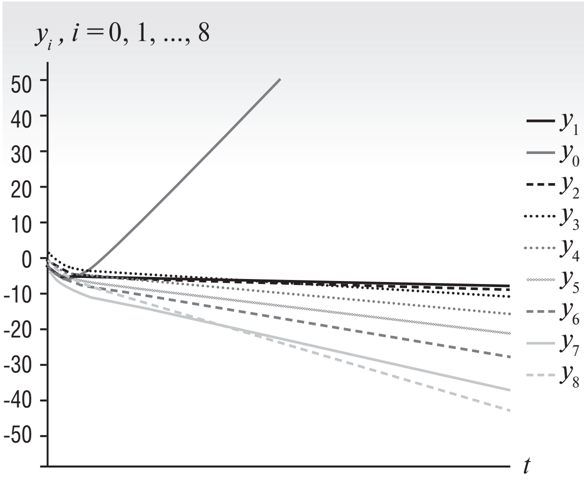

Fig. 5. Dynamics of the number of involved roads on railway tracks with increase of value of parameter c_0 (multiple increase)

Let us proceed to the study of the dependence of the growth (fall) of the number of involved roads on the railway tracks on the parameter c. This parameter determines the intensity of the shipment of cargo from any intermediate station with the number i = 1, ..., m on the railway tracks. Sending cargo to the railway track is carried out if the number of involved roads at the station is greater than the value Δ, that determines the capacity of the station. According to (5), this condition is satisfied at stations with numbers i = 0, ..., $\overline{m}$. Thus, the station with number i = 0, ..., $\overline{m}$ sends cargo to the railway track with number i = 0, ..., $\overline{m}$. Let's remember that the value $\overline{m}$ depends on parameter c: with its increase the value $\overline{m}$ decreases to $\overline{m}$ = 1. Therefore, a small increase in parameter c which is not leading to reduction of value $\overline{m}$ leads to an increase in the growth rate of the number of involved roads on railway tracks with numbers i = 1, ..., $\overline{m}$ and to reduction of growth of the number of the involved roads on the railway track with number i =0. On railway tracks with numbers $i = \overline{m} + 1$, ..., m an increase in speed of fall of the number of involved roads is observed. At the same time as an increase in the growth rate of the number of involved roads on railway tracks with numbers i = 1, ..., $\overline{m}$, and increase in speed of fall of the number of involved roads on railway tracks with numbers $i = \overline{m} + 1, ..., m$ weakens at increase in number of a railway track.

If the increase in parameter c leads to the reduction of value $\overline{m}$, then the following trend is observed: in the process of an increase in parameter c gradually on railway tracks on which there was an increase in the growth rate of the number of involved roads there is a reduction of growth rate of the number of involved roads up to further falls, except for railway track to numbers $i=0.1$ (*Figure 6*). Thus, since some value of parameter c, on all railway tracks except zero and the first, there is a decrease in number of the involved roads. On zero and first railway tracks there is a growth of number of the involved roads, and the growth rate on the first increases (*Figure 7*).

In *Figure 6*, the value of the parameter c is increased to two ($\alpha = 10,\ a = 0.1,\ c_0 = 1,\ c = 2,\ d = 3$), and in *Figure 7* – to 60, with unchanged values of other parameters ($\alpha = 10,\ a = 0.1,\ c_0 = 1,\ c = 60,\ d = 3$).

Let's pass to a research of dependence of growth (fall) of the number of involved roads on railway tracks from parameter a. Let's remember that this parameter determines intensity of receipt of cargo by the second technology (from a railway track), and this technology is applied if the number of involved roads at the station are less than value Δ. According to (6),

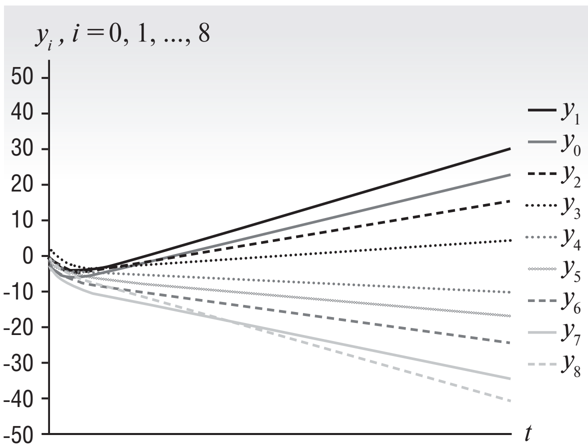

Fig. 6. Dynamics of the number of involved roads on railway tracks at increases of value of parameter c (double increase)

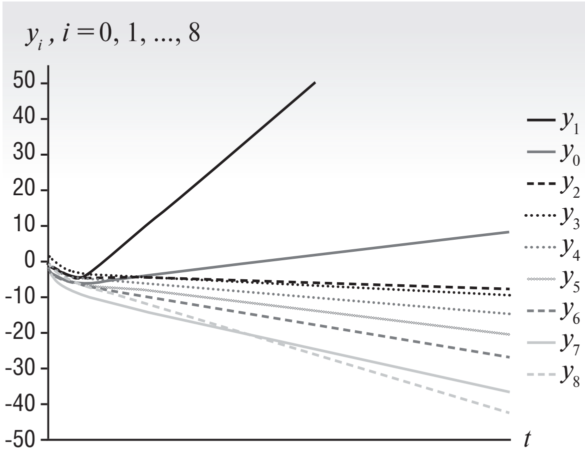

Fig. 7. Dynamics of the number of involved roads on railway tracks at increases of value of parameter c (multiple increase)

this condition is satisfied at stations with numbers $i = \overline{m} + 1, ..., m + 1$. As was stated above, within the second technology the station with number $i + 1$ accepts cargo from a railway track with number i. Let's remember that the value $\overline{m}$ depends on parameter a: with its increase the value $\overline{m}$ increases to $\overline{m} = m$. Therefore, a small increase in parameter a, which is not leading to increase in value $\overline{m}$, leads to an increase in speed of fall on railway tracks with numbers $i = \overline{m}, ..., m$. On railway tracks with numbers $i = 0, ..., \overline{m} - 1$ an increase in the growth rate of the number of involved roads is observed. At the same time as increase in speed of fall on railway tracks with numbers $i = \overline{m}, ..., m$, and increase in growth rate of number of involved roads on the previous railway tracks weakens with the reduction of the number of railway track. If the increase in parameter a leads to an increase in value $\overline{m}$, then the following trend is observed: in the process of increases in parameter a gradually on railway tracks on which there was an increase in the speed of fall in the number of involved roads there is a reduction of speed of fall in the number of involved roads up to further growth, except for the last railway track to number $i = m$ (*Figure 8*). Thus, since some value of parameter a, on all railway tracks except the last, there is a growth of number of the involved roads. On the last railway track, there is a falling

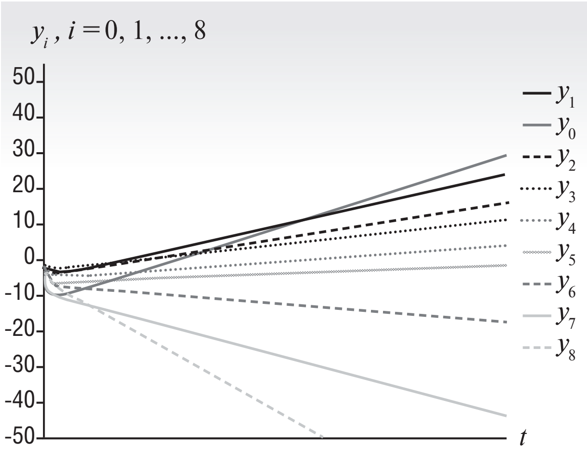

Fig. 8. Dynamics of the number of involved roads on railway tracks with increases of value of parameter ***a*** (fivefold increase)

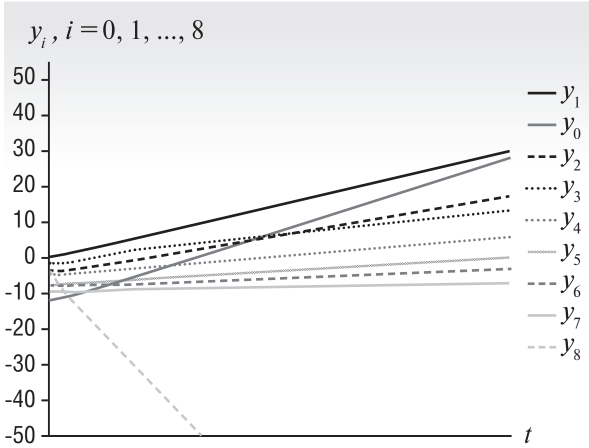

Fig. 9. Dynamics of the number of involved roads on railway tracks with increases of value of parameter ***a*** (multiple increase)

number of involved roads, and the speed of fall increases (*Figure 9*).

In *Figure 8*, the value of parameter a is increased to 0.5 ($\alpha = 10,\ a = 0.5,\ c_0 = c = 1,\ d = 3$), and in *Figure 9* — to 10, with unchanged values of other parameters ($\alpha = 10,\ a = 10,\ c_0 = 1, c = 60,\ d = 3$).

At last, we investigated the dependence of growth (fall) of volume of railway tracks on parameter α. Let's remember that change of this parameter does not change value $\bar{m}$. According to the research conducted in [27] since timepoint $\bar{t}$, an increase in parameter α leads to a reduction as differences $(z_i - \Delta)$, $i = 0, ..., \bar{m}$ and $(\Delta - z_i)$, $i = \bar{m} + 1, ..., m$. Thus, an increase in parameter α leads to reduction of the growth rate on railway tracks with numbers $i = 0, ..., \bar{m} - 1$, and to reduction of speed of fall in the number of involved roads on railway tracks with numbers $i = \bar{m} + 1, ..., m$. The same impact is made by an increase in this parameter and number of involved roads on a railway track with number $i = \bar{m}$, only with the difference that the number of involved roads on this railway tracks can both grow, and fall, or not change. For example, the dynamics at the following values of parameters ($\alpha = 30,\ a = 0.1,\ c_0 = c = 1,\ d = 3$) is given in *Figure 10.*

Thus, an increase in parameter α can reduce both growth, and fall of the number of involved roads on all railway tracks.

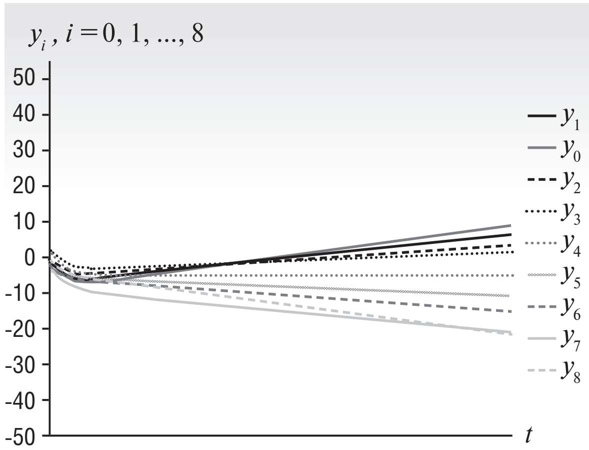

Fig. 10. Dynamics of the number of involved roads on railway tracks with increase of value of parameter α

Conclusion

This article is devoted to research into the dynamics of capacities of railway tracks in a model for the organization of cargo transportation between two node stations. Earlier in works [24—27] the dynamics of capacities of stations was investigated (number of involved roads on them). In the research carried out, it was supposed that capacities of railway tracks are unlimited and for that reason observation of their dynamics was not made. In this work, the system of differential equations describing the number of involved roads on railway tracks is presented and investigated. As it appeared, from some time point, the number of involved roads on all railway tracks, except one, either increases, or decreases. At the same time, the quantity of railway tracks both with increasing, and with decreasing number of involved roads depends on a number of parameters of the model. The dependence of the growth rate and fall of number of the involved roads on model parameters is investigated. We revealed the parameter, which if increased makes it possible to achieve simultaneous reduction of both growth rate and speed of fall in the number of involved roads on all railway tracks. This parameter characterizes the intensity of interaction of the neighboring stations within the first technology of the organization of freight traffic. ■

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