Estimation of resources required for restoring a system of computer complexes with elements of different significance

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Abstract

Large distributed information systems (LDIS) are the basis for digitization of production processes in industry, transport and public administration. Organization of their engineering servicing (ES) for timely restoration in case of failure is a topical issue of scientific research. LDIS consists of computer complexes which include both major and additional elements. The literature provides no solution which allows us to define the engineering servicing resources (ESR), considering the variable significance of elements of computer complexes. The task was first set and solved in this publication.

For solution of this task, we applied the mean dynamic method. This method was chosen because it makes it possible to obtain a system of differential equations for describing the change over time of the mean number of elements in different states.

Analysis of the differential equations system solution allowed us to find analytical expressions for determining ESR – the number of staff and the number of spare elements at which the mean number of computer complexes in perfect state reaches its maximum. The results are applicable when calculating the ESR of real LDIS. They can also serve in simulation modeling as initial approximations of the optimal volume of ESR if it necessary to take into account the specific features of the system. In addition, the solution of differential equations makes it possible to solve the problem of optimizing the resources for servicing the LDIS according to economic criteria, when the costs of staff and spare elements are comparable with the income from operating the computer complexes.

Key words: distributed information systems; engineering servicing; dependability; queuing theory; mean dynamics method.

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Introduction

Large distributed information systems (LDIS) are used for production tasks in various industries and public administration [1–6]. An example of such a system is the system for tracking the location of railway carriages and containers on the Railways of Russia and CIS countries. It is a network including the central system and peripheral computer complexes (CC). Public administration systems created in the framework of the Electronic Russia program also include CC [7]. The dependability of such systems must be high. In particular, if they ensure the process paramount for this industry [8, 9]. Thus, railways of Russia and CIS countries apply integrated systems of automated traffic control. Such systems ensure execution of the trains' working schedule, taking into account the actual capacity of different sections [10, 11]. The dependability of such systems is defined by reliability measures and restoration measures. Restoration measures depend on how rationally engineering servicing (ES) is organized. In practice, to restore LDIS promptly, it is reasonable to create special ES centers (ESC). ESC centrally maintain particular regions, using substitutions and subsequent repairing of the elements under stationary conditions as the main restoration method. To perform its tasks, ES must have adequate labor and material resources – engineering servicing resources (ESR). If the volume of these resources is small, the system will accumulate inoperative equipment. At a minimum, the efficiency of the system will be significantly reduced. If they are too large, this adds unnecessary costs.

Each of the systems mentioned above consists of computer complexes. Failure of a computer complex does not lead to the failure of the system, but reduces its performance. In [12], the task of ESR definition is solved for LDIS consisting of computer complexes, in which all elements are necessary to the same extent for it to have an up state. The metering system for electricity consumption is an example of such a system. However, many systems consist of computer complexes in which some elements (1-st type elements) are major, and others (2-nd type elements) are not. Failure of the 1-st type elements causes computer complex failure. Failure of the 2-nd elements makes the complex imperfect – its efficiency of performance falls, but it does not lead to computer complex failure. In the literature, there is no solution for ESR definition for such systems. This publication is offered to fill this gap. The research was prompted by the fact that most actual LDIS belong to this very type.

The aim of this paper is to increase the economic operating efficiency of LDIS thanks to the rational choice of the ESR.

The article has the following structure. In the first section, there is a mathematical formulation of the problem. The notations used are also entered. The second section consists of three subsections. In the first, differential equations are derived to describe the dynamics of changes in the average numbers of elements in different states during the operation of the LDIS. Much attention here is paid to the derivation of the values of the intensity of transitions of an element from one state to another. It is shown that these intensities are generally not constant. By virtue of this fact, differential equations are not linear. In the second subsection, formulas are derived for determining the volume of the ESR at which the average number of computer complexes in a perfect state reaches its maximum. The third subsection provides an example of the calculation of the ESR. The third section of the article indicates which of the restrictions made when setting the task are most significant. In addition, we explain why it is impossible to apply the classical method of calculating queuing systems based on the Kolmogorov equations for an analytical solution of the problem posed. The fourth section indicates when it is advisable to apply the formulas for calculating ESR obtained in the work, and when it is necessary to solve a rigid system of nonlinear differential equations of mean dynamics numerically. The Conclusion indicates which new scientific results have been obtained in this work.

1. Formulation of the problem

Let us consider LDIS consisting of N computer complexes, each consisting of two elements. One of them (the 1-st type element) is compulsory for the up state of the complex, failure of another (the 2-nd type element) only reduces its performance. Failure of one computer complex does not lead to failure of the whole system, but only reduces its performance.

Let us introduce the following indices for elements of 1-st and 2-nd types: mean operating time to failure are $\overline{T}_0^{(1)}$, $\overline{T}_0^{(2)}$; mean time of substitution are $\overline{T}_{\text{sub}}^{(1)}$, $\overline{T}_{\text{sub}}^{(2)}$; mean time of repair are $\overline{T}_{\text{rep}}^{(1)}$, $\overline{T}_{\text{rep}}^{(2)}$. The mean operating time to failure of the 1-st and 2-nd type elements is longer than the total time required for their substitution and repair. To ES of the LDIS, an ESC is created. Replacement of the elements is performed by the team of couriers (r persons). The repair performed by another team from which special groups for 1-st and 2-nd type elements allocated, numbered $R^{(1)}$ and $R^{(2)}$ respectively. At the initial time point, there are $n^{(1)}$ of spare 1-st type elements, and $n^{(2)}$ 2-nd type elements.

Required to determine: the number of staff and spare elements under which the average number of perfect complexes will be stable at the maximum value.

2. Solution method

2.1. The model of restoration

Let us suppose random values: operating time to failure; time substitution; times of repair of 1-st and 2-nd type elements are exponential and distributed with the following parameters: $\lambda_0^{(1)}, \lambda_0^{(2)}, \lambda_{\text{sub}}^{(1)}, \lambda_{\text{sub}}^{(2)}, \lambda_{\text{rep}}^{(1)}, \lambda_{\text{rep}}^{(2)}$. The value of each parameter is the inverse to the mean time. Then, to describe the change in the mean numbers of the elements in different conditions, we can apply the mean dynamics method [13].

In this case, the graph of system element conditions consists of two sub-graphs shown in *Figure 1*. Unlike the system of independent elements, these sub-graphs are not identical.

The first sub-graph describes possible conditions of the 1-st type element whose failure leads to failure of the computer complex in which it is included. The nodes of this graph have the following meaning:

$S_1^{(1)}$ – 1-st type element is in perfect state and functions as a part of a computer complex;

$S_2^{(1)}$ – 1-st type element has failed and is expecting a substitution;

$S_3^{(1)}$ – 1-st type element has failed, delivered to the ESC, is awaiting repair or is being repaired;

$S_4^{(1)}$ – 1-st type element is in a perfect state and is in the storehouse of the ESC.

The sub-graph which identifies the condition of the 2-nd type element has, unlike the first sub-graph, not four but five nodes. Four of them are: $S_1^{(2)}$, $S_2^{(2)}$, $S_3^{(2)}$, $S_4^{(2)}$ which correspond to the conditions of the 2-nd type elements, similar to $S_1^{(1)}$, $S_2^{(1)}$, $S_3^{(1)}$, $S_4^{(1)}$. The fifth node shows the $S_5^{(2)}$ condition: the 2-nd type element is perfect, is a part of the computer complex, but does not operate due to failure of the 1-st type element. It is the possibility of such conditions that reflects interdependence between the elements.

Change in the number of the elements is described by the solution of the differential equation set:

$$\begin{cases} \dfrac{dm_1^{(1)}(t)}{dt} = -\lambda_{12}^{(1)} m_1^{(1)}(t) + \lambda_{41}^{(1)} m_4^{(1)}(t), \\ \dfrac{dm_2^{(1)}(t)}{dt} = -\lambda_{23}^{(1)} m_2^{(1)}(t) + \lambda_{12}^{(1)} m_1^{(1)}(t), \\ \dfrac{dm_3^{(1)}(t)}{dt} = -\lambda_{34}^{(1)} m_3^{(1)}(t) + \lambda_{23}^{(1)} m_2^{(1)}(t), \\ \dfrac{dm_4^{(1)}(t)}{dt} = -\lambda_{41}^{(1)} m_4^{(1)}(t) + \lambda_{34}^{(1)} m_3^{(1)}(t), \\ m_1^{(1)}(t) + m_2^{(1)}(t) + m_3^{(1)}(t) + m_4^{(1)}(t) = \\ = N + n^{(1)}, \\ \dfrac{dm_1^{(2)}(t)}{dt} = -[\lambda_{12}^{(2)} + \lambda_{15}^{(2)}] m_1^{(2)}(t) + \\ + \lambda_{41}^{(2)} m_4^{(2)}(t) + \lambda_{51}^{(2)} m_5^{(2)}(t), \\ \dfrac{dm_2^{(2)}(t)}{dt} = -\lambda_{23}^{(2)} m_2^{(2)}(t) + \lambda_{12}^{(2)} m_1^{(2)}(t), \\ \dfrac{dm_3^{(2)}(t)}{dt} = -\lambda_{34}^{(2)} m_3^{(2)}(t) + \lambda_{23}^{(2)} m_2^{(2)}(t), \\ \dfrac{dm_4^{(2)}(t)}{dt} = -\lambda_{41}^{(2)} m_4^{(2)}(t) + \lambda_{34}^{(2)} m_3^{(2)}(t), \\ \dfrac{dm_5^{(2)}(t)}{dt} = -\lambda_{51}^{(2)} m_5^{(2)}(t) + \lambda_{15}^{(2)} m_1^{(2)}(t), \\ m_1^{(2)}(t) + m_2^{(2)}(t) + m_3^{(2)}(t) + m_4^{(2)}(t) + \\ + m_5^{(2)}(t) = N + n^{(2)}, \end{cases} \qquad (1)$$

with the initial conditions

$$\begin{aligned} & m_1^{(1)}(0) = m_1^{(2)}(0) = N; \\ & m_4^{(1)}(0) = n^{(1)}; \quad m_4^{(2)}(0) = n^{(2)}; \\ & m_2^{(1)}(0) = m_2^{(2)}(0) = m_3^{(1)}(0) = \\ & = m_3^{(2)}(0) = m_5^{(2)}(0) = 0. \end{aligned} \qquad (2)$$

There is a correlation between the number of elements of different types. The intensity of element transition from one condition to another depends on the distribution of the elements by conditions. The type of the expressions which define the values of $\lambda_i^{(1)}(t)$ ($i = 1, 2, 3, 4$) and $\lambda_j^{(2)}(t)$ ($j = 1, 2, 3, 4, 5$) will be different in different correlations between priorities of the elements. Two cases are typical:

✦ priorities of 1-st and 2-nd type elements are the same;

✦ 1-st-type elements are replaced out of queue, whereas their capacity is the compulsory condition of the complexes' functioning.

Let us consider the second case, when the priority for maintenance of the 1-st type elements is higher than for the 2-nd type elements.

Let us define the intensity of the element transition from one condition to another. Some of these intensities are permanent, and some depend on the number of elements which are in other conditions. As a result, system (1) is non-linear. The numbers of elements in different conditions are unknown. Following the Diner principle, we will replace unknown numbers of elements in different conditions with their mean values, and later on, we will use the expressions "numbers of elements" and "mean numbers of elements" as synonyms. Let us start with $\lambda_{12}^{(1)}(t)$ – the intensity of transition of the 1-st type element $S_1^{(1)}$ condition to $S_2^{(1)}$ condition. It is permanent. Indeed, if at the instant of time t, $m_1^{(1)}(t)$ the 1-st type elements function, the total intensity of their intensities will count

$$\Lambda^{(1)}(t) = \lambda_0^{(1)} m_1^{(1)}(t), \qquad (3)$$

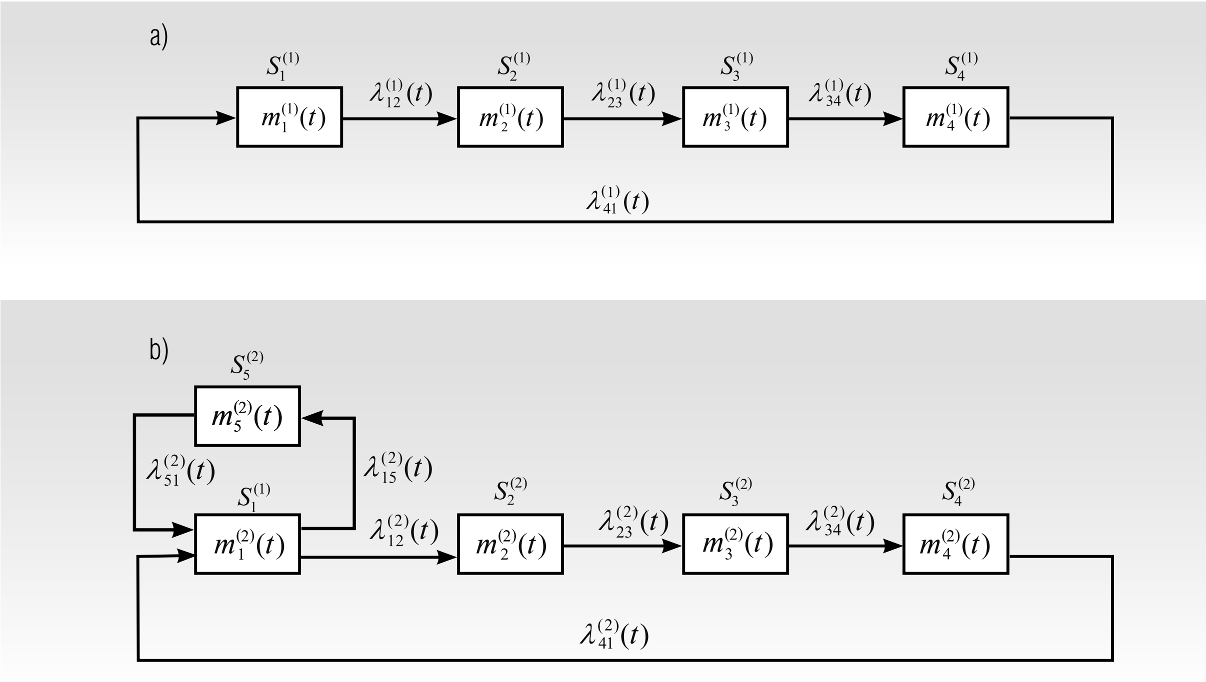

Fig. 1. Graph of states for (a) 1–st type elements, (b) 2–nd type elements

which, counting on one element, gives us:

$$\lambda_{12}^{(1)}(t) = \frac{\Lambda^{(1)}(t)}{m_1^{(1)}(t)} = \lambda_0^{(1)}. \quad (4)$$

Let us define $\lambda_{23}^{(1)}(t)$: this intensity of the transition will not be constant. Indeed, let's assume that in t instant of time, $m_2^{(1)}(t)$, the 1-st type elements are inoperative and expecting a courier to be replaced with the spare ones. Two variants of the courier's actions are possible here:

✦ to deliver the elements to the ESC, regardless of the available spare elements, and then install the spare elements as they come from repair;

✦ if there are no spare elements, to expect receipt of the fixed elements and do not deliver to the ESC without replacement.

The first variant of the courier's actions allows us, as a rule, in some way to reduce the demand of spare elements, as compared to the second one, so let us consider it in more detail. Here the intensity of the $\lambda_{23}^{(1)}(t)$ transition depends only on correlation between the number of couriers r, and the elements, in $S_2^{(1)}$ condition. In fact, if $r \geq m_2^{(1)}(t)$, then the total intensity of the transition from $S_2^{(1)}$ into $S_3^{(1)}$ will equal

$$\Lambda_{23}(t) = \lambda_{\text{sub}}^{(1)} m_2^{(1)}(t), \quad (5)$$

which, counting on one element, gives us:

$$\lambda_{23}^{(1)}(t) = \lambda_{\text{sub}}^{(1)}. \quad (6)$$

If $r < m_2^{(1)}(t)$, then the total intensity of transition from $S_2^{(1)}$ to $S_3^{(1)}$ will equal

$$\Lambda_{23}(t) = \lambda_{\text{sub}}^{(1)} r, \quad (7)$$

which, counting on one element, gives us:

$$\lambda_{23}^{(1)}(t) = \frac{\lambda_{\text{sub}}^{(1)} r}{m_2^{(1)}(t)}. \quad (8)$$

In a similar way, it is possible to define the intensity of the repair counting on one element:

$$\lambda_{34}^{(1)}(t) = \begin{cases} \lambda_{\text{rep}}^{(1)}, \ R \geq m_3^{(1)}(t); \\ \dfrac{\lambda_{\text{rep}}^{(1)} R^{(1)}}{m_3^{(1)}(t)}, \ R < m_3^{(1)}(t). \end{cases} \quad (9)$$

The intensity of the transition of the 1-st type element from $S_4^{(1)}$ to the $S_1^{(1)}$ condition is defined by correlations between $m_{\text{TR}}^{(1)}(t)$ – number of 1-st type elements, which must be sent at t moment for installation into the system, the number of couriers r and the number of spare elements available in the storehouse. The value of $m_{\text{TR}}^{(1)}(t)$ equals the difference between the total number of 1-st type elements in the operative system N and the number of operative 1-st type elements $m_1^{(1)}(t)$:

$$m_{\text{TR}}^{(1)}(t) = N - m_1^{(1)}(t). \quad (10)$$

At some instances of time, the demand of elements cannot coincide with the number of those expecting the replacement $m_2^{(1)}(t)$, as the couriers take away the failed elements for ESC also when there is nothing to replace them with. As a result, the following correlation is true:

$$m_{\text{TR}}^{(1)}(t) = N - m_1^{(1)}(t) \neq m_2^{(1)}(t). \quad (11)$$

It will take place if there are not enough spare elements or the free couriers. Taking into account the latest, we'll get the following expressions for the intensity of the transition $\lambda_{41}^{(1)}(t)$:

$$\lambda_{41}^{(1)}(t) = \frac{\lambda_{\text{sub}}^{(1)}, \min\left\{\left[N - m_1^{(1)}(t)\right], r, m_4^{(1)}(t)\right\}}{m_4^{(1)}(t)}. \quad (12)$$

Now, let's define the intensity of transitions for the 2-nd type elements. The distinctive feature of the 2-nd type element is that it can be operative, be a part of the complex, but not function due to failure of a 1-st type element.

In a similar way as was done for the 1-st type element, we can show that

$$\lambda_{12}^{(2)}(t) = \lambda_0^{(2)}. \quad (13)$$

The intensity of transition of a 2-nd type element from $S_2^{(2)}$ to $S_3^{(2)}$ condition, if the priority for the replacement is higher for the 1-st type element, depends on the correlation of the 2-nd type elements which must be delivered to the ESC – $m_2^{(2)}(t)$ and the number of couriers $r_{\text{free}}(t)$ not busy working on the 1-st type elements.

It is easy to make sure that

$$r_{\text{free}}(t) = r - m_{\text{TR}}^{(1)}(t) = r - \left[N - m_1^{(1)}(t)\right] = \\ = r + m_1^{(1)}(t) - N. \quad (14)$$

Respectively

$$\lambda_{23}^{(2)}(t) = \begin{cases} 0, \; r_{\text{free}} \leq 0\,; \\ \dfrac{\lambda_{\text{sub}}^{(2)}, \min\left\{r_{\text{free}}(t), m_2^{(2)}(t)\right\}}{m_4^{(2)}(t)}, \; r_{\text{free}}(t) > 0\,. \end{cases} \quad (15)$$

The intensity of the transition from repair to the storehouse for a 2-nd type element:

$$\lambda_{34}^{(2)}(t) = \begin{cases} \lambda_{\text{rep}}^{(2)}, \quad m_3^{(2)}(t) \leq R^{(2)}; \\ \dfrac{\lambda_{\text{rep}}^{(2)} R}{m_3^{(2)}(t)}, \; m_3^{(2)}(t) > R^{(2)}. \end{cases} \quad (16)$$

The intensity of $\lambda_{41}^{(2)}(t)$ depends on correlations between the numbers of free couriers $r_{\text{free}}(t)$, spare 2-nd type elements in the storehouse $m_4^{(2)}(t)$, and demand for spare 2-nd type elements which must be installed in the system – $m_{\text{TR}}^{(2)}(t)$.

The value of $m_{\text{TR}}^{(2)}(t)$, unlike $m_{\text{TR}}^{(1)}(t)$, does not equal the difference between the total number of complexes in the system and the number of operative elements of this type. It is related to the fact that 2-nd type elements cannot function due to the failure of a 1-st type element, therefore

$$m_{\text{TP}}^{(2)}(t) = m_1^{(1)}(t) - m_1^{(2)}(t). \quad (17)$$

Taking all of this into account, we will finally obtain

$$\lambda_{41}^{(2)}(t) = \begin{cases} 0, \; r_{\text{free}} \leq 0\,; \\ \dfrac{\lambda_{\text{sub}}^{(2)}, \min\left\{r_{\text{free}}(t), m_{\text{TR}}^{(2)}(t), m_4^{(2)}(t)\right\},}{m_4^{(2)}(t)} \\ \quad r_{\text{free}}(t) > 0\,. \end{cases} \quad (18)$$

Now we derive the formula which allows one to define $\lambda_{15}^{(2)}(t)$, the failures of the 1-st type elements, bring 2-nd type elements into $S_5^{(2)}$ condition, with total intensity

$$\Lambda_5(t) = \lambda^{(1)} m_1^{(1)}(t). \tag{19}$$

Counting on one 2-nd type element in the $S_1^{(1)}$ condition, we will obtain

$$\lambda_{15}^{(2)}(t) = \frac{\lambda_0^{(1)} m_1^{(1)}(t)}{m_1^{(2)}(t)}. \tag{20}$$

The intensity of transition $\lambda_{51}^{(2)}(t)$ equals the ratio of the total intensity of transitions of the 1-st type elements into the operative state to the number of 2-nd type elements in the $S_5^{(2)}$ condition:

$$\lambda_{51}^{(2)}(t) = \frac{\lambda_{\text{sub}}^{(1)} \min\left\{\left[N - m_1^{(1)}(t)\right], r, m_4^{(1)}(t)\right\}}{m_5^{(2)}(t)}. \tag{21}$$

By the numerical solution of system (1) at the found values of intensity, which are coefficients of the equations, it is possible to determine the distribution of elements by states at any number of couriers, repairmen and spare elements. The equations of system (1) are rigid [14] because the time between failures in modern computer complexes can be several orders more than average times of replacement and repair. At the same time, the complexity of calculations does not go beyond the methods realized in MathCAD [14, 15].

2.2. Estimating required engineering servicing resources

Now we define the sufficient ESR, in which, the mean number of perfect complexes of the system will be stable to maximum. This task amounts to defining the number of staff and spare elements in which the solution of the system will not go beyond the low-queue region.

Boundaries of the low-queue region, in this case, can be set by the inequality set:

$$\begin{cases} r_{\text{free}}(t) \geq m_{\text{TR}}^{(1)}(t) + m_{\text{TR}}^{(2)}(t), \\ R^{(1)} \geq m_3^{(1)}(t), \\ R^{(2)} \geq m_3^{(2)}(t), \\ m_4^{(1)}(t) \geq m_{\text{TR}}^{(1)}(t), \\ m_4^{(2)}(t) \geq m_{\text{TR}}^{(2)}(t). \end{cases} \tag{22}$$

In the low-queue region, the coefficients of the system (1) take the values:

$$\begin{aligned} &\lambda_{12}^{(1)} = \lambda_0^{(1)};\ \lambda_{23}^{(1)} = \lambda_{\text{sub}}^{(1)};\ \lambda_{34}^{(1)} = \lambda_{\text{rep}}^{(1)}; \\ &\lambda_{41}^{(1)} = \frac{\lambda_{\text{sub}}^{(1)}(N - m_1^{(1)})}{m_4^{(1)}}; \\ &\lambda_{12}^{(2)} = \lambda_0^{(2)};\ \lambda_{23}^{(2)} = \lambda_{\text{sub}}^{(2)};\ \lambda_{34}^{(2)} = \lambda_{\text{rep}}^{(2)}; \\ &\lambda_{41}^{(2)} = \frac{\lambda_{\text{sub}}^{(2)}(m_1^{(1)} - m_1^{(2)})}{m_4^{(2)}}; \\ &\lambda_{15}^{(2)} = \frac{\lambda_0^{(1)} m_1^{(1)}}{m_1^{(2)}};\ \lambda_{51}^{(2)} = \frac{\lambda_{\text{sub}}^{(1)}(N - m_1^{(1)})}{m_5^{(2)}}. \end{aligned} \tag{23}$$

Solution of the set (1) in the low-queue region (22) is being stabilized to the limiting values $m_i^{(1)}$ and $m_j^{(2)}$. To find them, it is enough to solve the algebraic system from (1) by substitution of coefficient values into it (23) and replacement by zero of all the derivatives from the numbers of conditions by the time. Having solved it, we finally obtain:

$$\begin{aligned} &m_1^{(1)} = K_{\text{af}}^{(1)} N; \\ &m_2^{(1)} = \left(1 - K_{\text{af}}^{(1)}\right) N; \\ &m_3^{(1)} = \frac{\overline{T}_{\text{rep}}^{(1)}}{\overline{T}_0^{(1)} + \overline{T}_{\text{sub}}^{(1)}} N; \\ &m_4^{(1)} = n^{(1)} - m_3^{(1)}; \\ &m_1^{(2)} = K_{\text{af}}^{(1)} K_{\text{af}}^{(2)} N; \\ &m_2^{(2)} = K_{\text{af}}^{(1)} \left(1 - K_{\text{af}}^{(2)}\right) N; \\ &m_3^{(2)} = K_{\text{af}}^{(1)} \frac{\overline{T}_{\text{rep}}^{(2)}}{\overline{T}_0^{(2)} + \overline{T}_{\text{sub}}^{(2)}} N; \\ &m_4^{(2)} = n^{(2)} - m_3^{(2)}; \\ &m_5^{(2)} = N - m_1^{(1)}; \end{aligned} \tag{24}$$

where $K_{af}^{(1)}$, $K_{af}^{(2)}$ are maximum set values of the availability factor for the 1-st and 2-nd type elements:

$$K_{af}^{(1)} = \frac{\overline{T}_0^{(1)}}{\overline{T}_0^{(1)} + \overline{T}_{sub}^{(1)}},$$

$$K_{af}^{(2)} = \frac{\overline{T}_0^{(2)}}{\overline{T}_0^{(2)} + \overline{T}_{sub}^{(2)}}. \quad (25)$$

In the stabilized distribution of the element number, the boundaries of low-queue regions can be set by the inequality system:

$$\begin{cases} r_{free} \geq m_{TR}^{(1)} + m_{TR}^{(2)}, \\ R^{(1)} \geq m_3^{(1)}, \\ R^{(2)} \geq m_3^{(2)}, \\ m_4^{(1)} \geq m_{TR}^{(1)}, \\ m_4^{(2)} \geq m_{TR}^{(2)}. \end{cases} \quad (26)$$

Here r_{free}, $m_{TR}^{(1)}$, $m_{TR}^{(2)}$ are the limiting values, to which, in the low- queue region, the functions $r_{free}(t)$, $m_{TR}^{(1)}(t)$, $m_{TR}^{(2)}(t)$ go, in $t \to \infty$.

Let us define the non-redundant set of resources in which inequalities (26) will be performed. This set is sufficient to ensure stabilization of the mathematical expectation for the number of operative complexes in which the two-element function goes to its maximum value $t \to \infty$.

To do this, we define the $m_{TR}^{(1)}$ and $m_{TR}^{(2)}$ values. Taking into account (10), (17), (26), we will obtain:

$$m_{TR}^{(1)} = N - m_1^{(1)} = \left(1 - K_{af}^{(1)}\right) N;$$

$$m_{TR}^{(2)} = K_{af}^{(1)} \left(1 - K_{af}^{(2)}\right) N. \quad (27)$$

The second and the third inequalities of the system (26) are performed in all $R^{(1)} \geq R_0^{(1)}$ and $R^{(2)} \geq R_0^{(2)}$, where:

$$R_0^{(1)} = \left\lceil \frac{\overline{T}_{rep}^{(1)}}{\overline{T}_0^{(1)} + \overline{T}_{sub}^{(1)}} N \right\rceil,$$

$$R_0^{(2)} = \left\lceil K_{af}^{(1)} \frac{\overline{T}_{rep}^{(2)}}{\overline{T}_0^{(2)} + \overline{T}_{sub}^{(2)}} N \right\rceil. \quad (28)$$

By substituting (27) in the first inequality of the system (26), we'll find that the first inequality of the system (26) is performed in $r \geq r_0$, where

$$r_0 = \left\lceil N(1 - K_{af}^{(1)} K_{af}^{(2)}) \right\rceil. \quad (29)$$

In a similar way, we can make sure that the condition of absence of a queue for replacement due to absence of the spare elements is performed when the initial number of spare 1st-type elements: $n^{(1)} > n_0^{(1)}$, and the number of spare 2nd-type elements: $n^{(1)} > n_0^{(1)}$, where:

$$n_0^{(1)} = \left\lceil \frac{\overline{T}_{rep}^{(1)} + \overline{T}_{sub}^{(1)}}{\overline{T}_0^{(1)} + \overline{T}_{sub}^{(1)}} N \right\rceil,$$

$$n_0^{(2)} = \left\lceil K_{af}^{(1)} \frac{\overline{T}_P^{(2)} + \overline{T}_{sub}^{(2)}}{\overline{T}_0^{(2)} + \overline{T}_{sub}^{(2)}} N \right\rceil. \quad (30)$$

Finally, we are ready to find the set of resources $v_0 = \left\{R_0^{(1)}, R_0^{(1)}, r_0, n_0^{(1)}, n_0^{(2)}\right\}$ which are necessary. This provides the mean number of computer complexes in a perfect state whose mean, in the term of time, is stabilized to maximum values

$$m_{ps} = m_1^{(1)} - m_2^{(2)} = K_{af}^{(1)} K_{af}^{(2)} N. \quad (31)$$

2.3. Example of the calculation of required engineering servicing resources

For example, the system uses 1000 computer complexes. Each of them includes a main computer unit (1-st type element) and a backup storage device for the data input (2-nd type element) which is optional. Their mean operating times to failure are: $T_0^{(1)} = 6000$ hours, and $T_0^{(2)} = 3500$ hours. The mean times of substitution: $T_{sub}^{(1)}$ = 12 hrs. and $T_{sub}^{(2)}$ = 14 hrs.; the mean time of repair: $T_{rep}^{(1)}$ = 72 hrs. and $T_{rep}^{(2)}$ = 48 hrs.

It is necessary to define the number of staff and spare elements required for restoration.

Using the formulas (28)–(30), we can obtain: $r_0 = 7$; $R_0^{(1)}=12$; $R_0^{(2)}=14$; $n_0^{(1)}=14$, $n_0^{(2)}=18$. In this set of ESR, the mean number of opera-tive computer complexes amounts to the maximum defined by the formula (31): $m_{ps} = 994$. If it is necessary to define if it is possible, for instance, to compensate the shortage of repair team by increasing the number of spare elements. It can be done by solving system (1) in different: $R^{(1)}$, $R^{(2)}$, $n^{(1)}$ and $n^{(2)}$. At the same time, the values indicated above, downrange up to the limit, in which it makes sense to change them.

3. Discussion

The analytical expressions for ESR calculation have been obtained. We will discuss how significant the restrictions are.

1. The assumption about the two elements of a computer complex was made to simplify the account and do not reduce the generality of the obtained results. They can be easily generalized for the case where a computer complex includes several 1-st type elements and several 2-nd type elements.

2. The formulated condition that the mean time to failure for any element is longer than the total time for its substitution and repair is significant for stabilization of the system (1) solution.

3. In this publication, the task has been solved by the mean dynamics method, which is related to approximate analytical methods of solving the task of mass maintenance. The mean dynamic method allows us to make equations of changes in expectations (mean) of numbers of the system elements being in different possible conditions. Owing to that, it is applicable for large systems, unlike precise methods based on Kolmogorov's equations describing changes in the probability of the state in the whole system in general [16–21]. Use of Diners's principle in defining the intensities of the transitions makes the results of the set solution (1) approximate. Respectively, the true time of stabilization may differ from the estimated time. However, for the low-queue region (26), this difference is insignificant.

4. The distribution of the time to failure, time of substitution, and time of replacement to the exponential law is significant. However, the mean dynamics method is known to show good results also in laws of distribution of the approximated by the exponential.

5. Only using simulation, it is possible to check how much the distribution law affect the required ESR volume [22, 23]. At the same time, it should be noted that we model a closed system of mass maintenance with large amount of entities [24]. Also, we should by using simulation define how shift work of the staff influences the ESR volume required for the systems considered in this publication.

4. Recommendations

Based on this study, we can develop the following recommendations:

✦ Formulas (28)–(30) should be applied for defining ESR volume when the loss from inoperativeness of a computer complex is much higher than the costs for the staff salary and cost of spare elements. This typical situation is true for control systems in railway transport.

✦ If the ESR-related costs are comparable to the profit from the functioning of an operative computer complexes in the system, we set the goal of defining ESR in which the profit will be maximal [25]. Such a situation occurs, for instance, in leasing the equipment. In this case, to define the profit for a period, we must solve the system (1).

✦ In solving (1), we must consider that, as a rule, this is a rigid system of non-linear differential equations, since the intensity of the failures is much lower than the intensity of substitution and repair. It is necessary to select the solution method correctly.

✦ Changing the initial conditions in solving the system (1), we can define how much it will take the ESC staff to restore the LDIS if, due to some reasons, it will accumulate inoperative computer complexes.

Conclusion

As a result of the study, the following new results were obtained.

We developed a mathematical model for restoration of LDIS which consists of computer complexes, all of which include elements with different significance for its performance. The model represents a system of non-linear differential equations. The system solution describes change over time in the number of computer complexes under different conditions including the most significant ones: in up state and in perfect state.

We have defined the number of staff and spare elements in which the expectation (mean) of operative computer complexes stabilizes in time up to their maximal values. These results can also be directly used to define resources for real LDIS maintenance. ■

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