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Using Fishburne's sequences in suitable modeling used for sample data*

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Abstract

This article deals with probabilistic and statistical modeling of managerial decision-making in the economy based on sample data for the previous periods of time. For better definition, the study is limited to Markowitz's models in the problem of finding an effective portfolio of the field in the third information situation. The third information situation is a widespread decision-making situation and is characterized by the fact that the decision-maker sets, according to his opinion, are a linear order relation on the components of an unknown probabilistic distribution of the states of the economic environment. Often, from the point of view of the decision-maker, the components of an unknown probability distribution of the states of the economic environment must satisfy a partially reinforced linear order relation. As a result, the use of traditional statistical estimates turns out to be impossible, while the following question arises, which is practically not studied in the scientific literature. In this case, what formulas should be used to find statistical estimates and, above all, estimates of unknown probabilities of the state of the economic environment? As an estimate of an unknown probability distribution, we proposed to use the Fishburne sequence that satisfies all available constraints, while corresponding to the opinion of the decision maker and the linear order relation given by him. Fishburne sequences are a generalization of the well-known Fishburne formulas. It is fundamentally important that any Fishburne sequence satisfies a simple linear order relation, and under certain conditions, a partially strengthened linear order relation. Particular attention is paid to the entropic properties of generalized Fishburne progressions, which represent the most important class of Fishburne sequences, as well as the use of generalized Fishburne progressions to take into account the opinion of the decision maker. Such a scheme for estimating an unknown probability distribution has been developed, which makes it possible to achieve the correctness of probabilistic and statistical modeling, as well as appropriate consideration of the opinion of the decision-maker, uncertainty and risk.

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Key words: Fishburne sequence; managerial decision-making; Markowitz model; linear order relation; entropy; Fishburne progression.

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Introduction

In game theory modeling, a special role is played by the statistical game [1], which is a game of two participants: a decision-maker (DM) who consciously chooses his behavior, and "nature" (the economic environment in the case of economic modeling), which randomly turns out to be in its possible states.

In the case of economic modeling, it is often necessary to establish what type of order relation is performed on a set of states of the economic environment which is characteristic of the third information situation (IS) [1, p. 13].

Modeling of the economy requires taking into account a number of specific features inherent in the economy, primarily uncertainty and economic risk (for example, [2]). Failure to take into account these features entails management inefficiency. The appropriateness and correctness of modeling is largely determined by the degree of consideration of uncertainty, economic risk and the opinion of the DM about the specifics of the decision-making situation.

Methods and models of probability theory and mathematical statistics are widely used in economic modeling. A classic example of probabilistic-statistical modeling of the economy is the portfolio theory, which began with the works of Harry Markowitz [3, 4].

According to Markowitz's approach, the rate of return (profitability) of an arbitrary asset / portfolio is characterized by a corresponding random variable (RV), the values of which are determined by the conditions in which the economic environment may be (in fact, the stock market). For the sake of certainty and convenience, we will assume that the set of possible states of the economic environment is finite. In this case, the probability distribution of the states of the economic environment $\mathbf{q} = \left(q_1; \ldots; q_n \right)$ is a vector whose components are non-negative numbers satisfying the normalization property, while the RV characterizing the rate of return of the selected asset is a discrete RV (DRV). In practice, the available sample data are used as possible values of these DRV, namely the previously observed values of the corresponding profit margins, while it is customary to use traditional point estimates as estimates of the numerical characteristics of these DRV, the values of which are calculated based on the use of a uniform law, i.e. a vector $\mathbf{q}^* = \left(q_1^*; \ldots; q_n^* \right)$, where $q_1^* = \ldots = q_n^* = \frac{1}{n}$, as it estimates of the probability distribution.

Suppose that the distribution $\mathbf{q} = \left(q_1; \ldots; q_n \right)$ satisfies a linear relation of order (LRO) of one type or another. The main types of LRO were studied by Peter Fishburn [5–7], (and also reviewed, for example, in [1, pp. 77–80]. Fishburne formulas and their generalizations, called Fishburne sequences in the article, are used to estimate the corresponding distributions. For example, in [8] the author proposed the use of Fishburne sequences to bring generalized models of the optimal portfolio search problem to the classical (traditional) Markowitz model.

The purpose of this study is to develop a scheme for constructing such an estimate of the probability distribution which makes it possible to achieve the correctness of probabilistic and statistical modeling, as well as to best take into account the opinion of the DM about the type of LRO, to which the elements of the set of states of the economic environment obey, including for cases where the use of traditional point estimates is impossible because the most characteristic estimate of the probability distribution should differ from the uniform law.

The main objectives of the study are to develop:

- concepts of suitable modeling of managerial decision-making in the economy based on sample data, in particular, portfolio decision-making in the field of the third IP;
- a method for estimating the probability distribution based on the use of Fishburne sequences, primarily generalized Fishburne progressions;
- schemes for constructing such an estimate of the probability distribution that best takes into account the opinion of the DM.

1. Formulation of the research task

We introduce the following designations: r_{ij} – the value of the i-th rate of return of the asset in conditions when the economic environment was in j-th state, $i=\overline{1,k}$, $j=\overline{1,n}$; x_i – the share of the asset in the portfolio, $i=\overline{1,k}$; $\mathbf{x}=(x_1;\ldots;x_k)$ – portfolio (more precisely its structure); R_i – DRV, characterizing the rate of i return of the asset, $i=\overline{1,k}$; $R_\mathbf{x}=\sum_{i=1}^{k}R_i\cdot x_i$ – DRV, characterizing the rate of return of the portfolio $\mathbf{x}$; $\mathbf{q}=(q_1;\ldots;q_n)$ – probability distribution of the states of the economic environment; $m_i=\mathbf{M}(R_i)=\sum_{j=1}^{n}r_{ij}\cdot q_j$, $m_\mathbf{x}=\mathbf{M}(R_\mathbf{x})$ – mathematical expectations of the corresponding DRV; $\sigma_i^2=\mathbf{D}(R_i)=\sum_{j=1}^{n}r_{ij}^2\cdot q_j-m_i^2$, $\sigma_\mathbf{x}^2=\mathbf{D}(R_\mathbf{x})$ – variance of the corresponding DRV; $c_{il}=\operatorname{cov}(R_i;R_l)=\sum_{j=1}^{n}r_{ij}\cdot r_{lj}\cdot q_j-m_i\cdot m_l$ – covariance between the specified DRV, $i=\overline{1,k}$, $l=\overline{1,k}$.

If the exact true values r_{i1}, ..., r_{in}, of the corresponding DRV R_i are known and the first IC takes place when the probabilities $(q_1;\ldots;q_n)$ of states are known, then the values of the numerical characteristics m_i, σ_i^2, c_{ij}, can be found by the above formulas, and the numerical charac teristics $m_\mathbf{x}$ and $\sigma_\mathbf{x}^2$ DRV $R_\mathbf{x}=\sum_{i=1}^{k}R_i\cdot x_i$ of the $\mathbf{x}$ portfolio are functions of fractions x_1, ..., x_k. The classical Markowitz model (the model of the problem of finding an effective portfolio in the field of the first IS) can be presented in the following form:

$$m_\mathbf{x}=\sum_{i=1}^{k}m_i\cdot x_i\to\max_\mathbf{x}, \tag{1}$$

$$\sigma_\mathbf{x}^2=\sum_{i=1}^{k}\sum_{l=1}^{k}c_{il}\cdot x_i\cdot x_l\to\min_\mathbf{x}, \tag{2}$$

$$\sum_{i=1}^{k}x_i=1, \tag{3}$$

$$x_i\geq 0,\ i=\overline{1,k}. \tag{4}$$

An effective portfolio (in the Markowitz model) is usually called a portfolio whose structure is a Pareto optimal solution to the problem (1)–(4).

The main task of the DM (investor) is to find the structure of the optimal portfolio, i.e. the structure of such an effective portfolio, which, according to the DM, has all the desired properties, first of all, the best combination of the values of its numerical characteristics.

Let us now assume the possible values r_{i1}, ..., r_{in}, of an arbitrary DRV R_i are known, and the probabilities q_1, ..., q_n are not known, then the numerical characteristics m_i, σ_i^2, c_{ij} are probability functions q_1, ..., q_n, and the numerical characteristics $m_\mathbf{x}$ and $\sigma_\mathbf{x}^2$ DRV $R_\mathbf{x}=\sum_{i=1}^{k}R_i\cdot x_i$ of the $\mathbf{x}$ portfolio are probability functions q_1, ..., q_n and fractions x_1, ..., x_k. This assumption corresponds to the approach used in practice when using available sample data, i.e. when the values r_{i1}, ..., r_{in} are used as the observed values of the *i*-th rate of return of the asset, while unknown values of numerical characteristics m_i, σ_i^2, c_{ij}, as a rule, evaluate them with traditional point estimates, i.e. the values of the numerical characteristics of the sample

$$\overline{r_i} = \frac{1}{n} \cdot \sum_{j=1}^{n} r_{ij}, \, i = \overline{1, k},$$

$$\sigma_i^{*2} = \overline{r_i^2} - \overline{r_i}^2 = \frac{1}{n} \cdot \sum_{j=1}^{n} r_{ij}^2 - \overline{r_i}^2, \, i = \overline{1, k},$$

$$c_{il}^* = \frac{1}{n} \cdot \sum_{j=1}^{n} r_{ij} \cdot r_{lj} - \overline{r_i} \cdot \overline{r_l}, \, i = \overline{1, k}, \, l = \overline{1, k},$$

accordingly. We emphasize, now r_{ij} – this is the value of the *i* rate of return of the asset observed in the period (moment) of time, and to calculate the values $\overline{r_i}$, σ_i^{*2}, c_{il}^*, a uniform law was used as an estimate of the probability distribution.

The use of traditional point estimates may contradict the opinion of the DM about the significance (in formativeness) of various points in time. The opinion of the DM about the significance of various moments of time must necessarily be reflected in the generalized Markowitz model: in this case, the system of constraints of the task of finding an effective portfolio may contain such constraints for possible values q_1, ..., q_n components of the probability distribution that the uniform law does not satisfy.

The generalized Markowitz model of the problem of finding an effective portfolio in the field of the third OF the simple gallop can be written in the following form [8]:

$$H(\mathbf{q}) = -\sum_{j=1}^{n} q_j \cdot \ln q_j \to \max_{\mathbf{q}}, \quad (5)$$

$$m_{\mathbf{x}} = \sum_{i=1}^{k} m_i \cdot x_i \to \max_{\mathbf{x}}, \quad (6)$$

$$\sigma_{\mathbf{x}}^2 = \sum_{i=1}^{k} \sum_{l=1}^{k} c_{il} \cdot x_i \cdot x_l \to \min_{\mathbf{x}}, \quad (7)$$

$$\sum_{i=1}^{k} x_i = 1, \quad (8)$$

$$x_i \geq 0, \; i = \overline{1, k}, \quad (9)$$

$$q_1 \leq q_2 \leq \ldots \leq q_n, \quad (10)$$

$$\sum_{j=1}^{n} q_j = 1, \quad (11)$$

$$q_j \geq 0, \; j = \overline{1, n}. \quad (12)$$

The ratios (10) reflect the essence of a simple LRO and the opinion of the DM that the situation that developed in a later period has greater significance i.e. has a more significant impact on the present and future than the situation that developed in an earlier period of time. Many researchers express this opinion, which certainly corresponds to the realities of the economy. So, V.K. Semenychev and E.V. Semenychev note that “when forecasting in conditions of rapidly changing socio-economic phenomena, the information of later time periods is more important, more significant than the information of earlier periods” [9, p. 60]. We emphasize that in the case of portfolio decisions in the field of the third IP, with the fairness of a simple LRO, it is possible to use traditional point estimates, since a uniform law satisfies a simple LRO.

The generalized Markovitz model of the problem of finding an effective portfolio in the field of the third IC, with the validity of a partially enhanced LRO, can be written in the following form [8]:

$$H(\mathbf{q}) = -\sum_{j=1}^{n} q_j \cdot \ln q_j \to \max_{\mathbf{q}}, \quad (13)$$

$$m_{\mathbf{x}} = \sum_{i=1}^{k} m_i \cdot x_i \to \max_{\mathbf{x}}, \quad (14)$$

$$\sigma_{\mathbf{x}}^2 = \sum_{i=1}^{k} \sum_{l=1}^{k} c_{il} \cdot x_i \cdot x_l \to \min_{\mathbf{x}}, \quad (15)$$

$$\sum_{i=1}^{k} x_i = 1, \quad (16)$$

$$x_i \geq 0, \; i = \overline{1, k}, \quad (17)$$

$$\begin{cases} q_2 \geq q_1, \\ q_3 \geq q_1 + q_2, \\ \ldots \\ q_n \geq q_1 + q_2 + \ldots + q_{n-1}, \end{cases} \tag{18}$$

$$\sum_{j=1}^{n} q_j = 1, \tag{19}$$

$$q_j \geq 0, \; j = \overline{1, n}. \tag{20}$$

The ratios (18) reflect the essence of the partially enhanced LRO and the opinion of the DM that socio-economic conditions are changing extremely rapidly, while the sample data under consideration are time series characterized by a high rate of change. The DM is obliged to adhere to such an opinion in cases when there is either a pre-crisis (crisis) situation, or a sharp growth of the economy (of the relevant sector of the economy), etc. [10]. If the DM believes that the probability distribution satisfies the partially enhanced LRO, then this means that, in the opinion of the DM, the significance of the current time period is not less than the total significance of all previous time periods. In this case, it is impossible to use traditional point estimates, because the uniform law does not satisfy the partially enhanced LRO.

This indicates the relevance and necessity of developing such a method of constructing an estimate of the probability distribution that best takes into account the opinion of the DM about the type of order ratio on a set of states of the economic environment, which is in fact his opinion about the type of LRO that the probability distribution should satisfy.

It can be argued that the above situations of portfolio decision-making are characterized by a statistical game: with tasks (1)–(4), (5)–(12), (13)–(20) a statistical game is connected, given by a matrix $\mathbf{R} = \mathbf{R}_{k \times n} = (r_{ij})$, where r_{ij} is the corresponding value of the i rate of return of the asset. Note that portfolio decision-making is possible by solving the corresponding game [11]: when certain requirements are met, the solution of the antagonistic game given by the matrix $\mathbf{R} = \mathbf{R}_{k \times n} = (r_{ij})$ allows you to find an effective case $\mathbf{x} = (x_1^*; \ldots; x_k^*)$, while its structure does not depend on the probability distribution.

2. Basic notion and definitions

Here are the definitions of LRO of the main types [1, p. 78]:

1) a simple LRO is an order relation given by inequalities of the form $q_1 \geq q_2 \geq \ldots \geq q_n$ or $q_1 \leq q_2 \leq \ldots \leq q_n$;

2) a partially enhanced LRO is an order relation given by inequalities of the form $q_j \geq q_{j+1} + \ldots + q_n$, $j = \overline{1, n-1}$ or $q_j \geq q_1 + \ldots + q_{j-1}$, $j = \overline{2, n}$;

3) an enhanced LRO is an order relation given by inequalities of the form

$q_{j+1} + \ldots + q_{j+\alpha(j)} \leq q_j \leq q_{j+1} + \ldots + q_{j+\alpha(j)} + q_{j+\alpha(j)+1}$,

$j = \overline{1, n-2}$, $\alpha(j) \in \{1; 2; \ldots; n-1-j\}$, or

$q_{j-\alpha(j)} + \ldots + q_{j-1} \leq q_j \leq q_{j-\alpha(j)-1} + q_{j-\alpha(j)} + \ldots + q_{j-1}$, $j = \overline{3, n}$, $\alpha(j) \in \{1; 2; \ldots; j-2\}$, where $\alpha(j)$ – given natural numbers taking values from the specified sets.

The Fishburne sequence (FS) will be called the sequence $(q_1; q_2; \ldots; q_j; \ldots)$ the elements of which are equal

$$q_j = \frac{a_j}{\sum_i a_i}, \forall j, \tag{21}$$

where the sequence $(a_1; a_2; \ldots; a_j; \ldots)$ generating this FS is a given monotonic sequence of non-negative numbers, the sum of which is a positive number [12, p. 132]. Further, for convenience, we will limit ourselves to the consideration of finite FS $(q_1; q_2; \ldots; q_n)$.

Externally, formula (21) coincides with the so-called third Fishburne formula. The third Fishburne formula is used to estimate the probability distribution when it must satisfy an enhanced LRO. In this case, the formulas for calculating probability estimates contain a parameter, while using formula (21) allows us to find a value of this parameter for which the values of probability estimates satisfy both the enhanced LRO and condition (3). Obviously, the set of all FS is significantly wider than the set of all sequences satisfying the enhanced LRO and condition (3).

In the author's publications (for example, [8, 10, 12–16]), the following properties of FS and their special cases are investigated:

1) *Fishburne arithmetic progression, given by the first Fishburne formula:*

$$q_j = \frac{2\cdot(n-j+1)}{n\cdot(n+1)}, j=\overline{1,n}; \qquad (22)$$

2) *Fishburne geometric progression, given by the second Fishburne formula:*

$$q_j = \frac{2^{n-j}}{2^n-1}, j=\overline{1,n}; \qquad (23)$$

3) *Fishburne's increasing arithmetic progression:*

$$q_j = \frac{2\cdot j}{n\cdot(n+1)}, j=\overline{1,n}; \qquad (24)$$

4) *the increasing geometric progression of Fishburne:*

$$q_j = \frac{2^{j-1}}{2^n-1}, j=\overline{1,n}; \qquad (25)$$

5) *generalized Fishburne arithmetic progression:*

$$q_j = \frac{1}{n}-\frac{(n-1)\cdot x}{2}+(j-1)\cdot x = \frac{2-n\cdot(n-2\cdot j+1)\cdot x}{2\cdot n}, j=\overline{1,n}; \qquad (26)$$

for which its difference x satisfies the inequality

$$|x| \le \frac{2}{n\cdot(n-1)};$$

6) *generalized Fishburne geometric progression:*

$$q_j = \frac{x-1}{x^n-1}\cdot x^{j-1} = \frac{1-x}{1-x^n}\cdot x^{j-1}, j=\overline{1,n}; \qquad (27)$$

for which its denominator is x satisfies the inequality $x > 0$.

Often, the use of formula (24) or (25), for example, when the index values represent time points, is preferable to the use of formula (22) or (23), respectively. Formulas (22)–(25) are used in a wide variety of studies, as evidenced by numerous publications (for example, [17–35]).

Uniform law, e.g. vector $\mathbf{q}^* = (q_1^*;...;q_n^*)$, where $q_1^* = ... = q_n^* = \frac{1}{n}$ is a special case of the generalized arithmetic Fishburne progression (for the difference $x = 0$), and the generalized Fishburne geometric progression (for the denominator $x = 1$).

3. Estimation of probability distribution based on the use of Fishburne sequences

Thus, generalized Markowitz models of the problem of finding an effective portfolio can be brought to the classical Markowitz model by using FS. The problem is how to make a correct estimate of the probability distribution. The proposed scheme for constructing a correct estimate of the probability distribution is based on the properties of FS, primarily on the properties of generalized Fishburne progressions.

The correctness of using formulas (26), (27) depends on the answers to such questions. 1. When does the generalized Fishburne progression satisfy a simple LRO? 2. When does the generalized Fishburne progression satisfy a partially enhanced LRO? 3. When does the generalized Fishburne progression satisfy the Gibbs–Jaynes principle?

Recall that the Gibbs–Jaynes principle considers the vector to be the most characteristic estimate of an unknown distribution $\mathbf{q}^* = \left(q_1^*;...;q_n^*\right)$ maximizing the entropy value $H(\mathbf{q}) = -\sum_{j=1}^{n} q_j \cdot \ln q_j$ when all constraints are met, e.g. constraints (3), (4) and, possibly, one or more constraints that, in the opinion of the DM, the probability distribution should satisfy. The entropy approach significantly enriches the tools of analysis and modeling of economic risk. Obviously, if only the constraints (3), (4) are met, then the maximum value of entropy $H\left(\mathbf{q}^*\right) = \ln n = \max_{\mathbf{q}} H(\mathbf{q})$ is achieved for a uniform law, e.g. for a vector $\mathbf{q}^* = \left(q_1^*;...;q_n^*\right)$, where $q_1^* = ... = q_n^* = \frac{1}{n}$. In particular, on the set of all generalized Fishburne progressions, the maximum value of entropy is also achieved for a uniform law.

Note that an arbitrary FS, and hence an arbitrary generalized Fishburne progression, satisfies the corresponding simple LRO.

If generalized arithmetic Fishburne progressions almost always do not satisfy partially enhanced LRO, then generalized geometric Fishburne progressions, depending on the value of their denominator, in some cases satisfy, and in other cases do not satisfy partially enhanced LRO.

Consequently, the above questions have trivial answers on the set of all generalized Fishburne arithmetic progressions. However, on the set of all generalized Fishburne geometric progressions, the answers to these two questions are not so obvious. The theorems containing these answers are given and proved in the monograph of A.V. Sigal, E.S. Remesnik [12, pp. 119–127]. The essence of these theorems is as follows: an arbitrary generalized geometric Fishburne progression $(q_1; q_2; ...; q_n)$ satisfies the corresponding partially amplified LRO if and only if the value of its denominator belongs to the set $x \in \left(0; \alpha_n\right] \cup \left[\beta_n; +\infty\right)$, where α_n – the root of the equation $x^n - 2 \cdot x + 1 = 0$ owned by $\alpha_n \in (0{,}5;\ 1]$, $\beta_n = \frac{1}{\alpha_n}$, at the same time $x^* = \alpha_n$ ($x^* = \beta_n$) – the only value of the denominator x progressions (27), maximizing the value of entropy $H(\mathbf{q})$ for all non-increasing (non-decreasing, respectively) generalized geometric Fishburne progressions.

One of the natural methods of solving problems (5)–(12) and (13)–(20) is their reduction to problem (1)–(4) by using the most characteristic estimate of the probability distribution, i.e. by using the vector $\mathbf{q} = (q_1; ...; q_n)$, maximizing the entropy value on the set of all FS satisfying the LRO of the corresponding type. Practically without loss of generality, instead of the set of all FS, we can limit ourselves to considering only generalized Fishburne progressions, the most important and very broad special case of FS.

The vector maximizing the entropy value on the set of all FS (and, consequently, on the set of all generalized Fishburne progressions), by definition always satisfying a simple LRO, is a uniform distribution, i.e. the vector $\mathbf{q} = (q_1; ...; q_n)$, где $q_1^* = ... = q_n^* = \frac{1}{n}$. Therefore, in the case of problem (5)–(12), the use of traditional point estimates is advisable, both from the standpoint of mathematical statistics and from the standpoint of the suitability of modeling.

And a vector maximizing the entropy value on the set of all non-decreasing generalized Fishburne geometric progressions $(q_1; q_2; ...; q_n)$, satisfying a partially enhanced LRO is the progression (27), whose denominator is equal to $x = \beta_n$, where $\beta_n = \frac{1}{\alpha_n}$, α_n the root of the equation $x^n - 2 \cdot x + 1 = 0$ belongs to $\alpha_n \in (0{,}5;\ 1]$.

The estimation of an unknown probability distribution and the use of this estimate can be carried out according to the following five-step scheme.

Step 1. Choosing the type of LRO, which, according to the DM, the distribution should satisfy $(q_1; q_2; ...; q_n)$ probabilities.

Step 2. Choosing the sequence (a_1; a_2; ...; a_n) non-negative numbers, which, according to the DM, it is advisable to use as a sequence generating FS with the desired properties. This sequence (a_1; a_2; ...; a_n) must satisfy the LRO of the selected type, while it can be a sequence whose elements form, for example, a certain progression of natural numbers (including a constant $a_1 = a_2 = ... = a_n= 1$, strictly monotone arithmetic or strictly monotone geometric progression), Fibonacci numbers or Mersenne numbers.

Step 3. Construction of the FS, which, according to the DM, it is advisable to use as an estimate of the probability distribution, i.e. calculation by formula (21) of the values of the FS components (q_1; q_2; ...; q_n), generated by the selected sequence (a_1; a_2; ...; a_n).

Step 4. Applying the constructed FS as an estimate of the probability distribution, and, in particular, calculating the values of the estimates of the corresponding numerical characteristics according to formulas for calculating these numerical characteristics of the DRV, in which the values of the corresponding elements of the constructed FS are used instead of the probability values.

Step 5. Choosing the optimal solution, i.e. choosing to implement such a solution that has the best combination of the calculated values of the estimates of the numerical characteristics under consideration.

Conclusion

Fishburne formulas for calculating point estimates of probability distributions are well known and widely used in theoretical and practical research. For a correct assessment of the probability distribution of the states of the economic environment, it is advisable to use Fishburne sequences (FS) generalizing Fishburne formulas: as an estimate of the probability distribution, it is advisable to use an FS that has all the desired properties and, in particular, satisfies the type of linear order relation (LRO), which, according to the decision maker (DM), the probability distribution should satisfy. The most important types of LRO are simple LRO and partially reinforced LRO.

The DM can evaluate the probability distribution and use this estimate according to the following scheme, first given in the article.

1. Choosing the type of LRO that the probability distribution should satisfy.
2. The choice of a sequence that is appropriate to use as a sequence generating FS.
3. Construction of the FS, which is advisable to use as an estimate of the probability distribution.
4. Application of the constructed FS as an estimate of the probability distribution.
5. Choosing the optimal solution, i.e. the choice for the implementation of such a solution that has the best combination of the calculated values of the estimates of the numerical characteristics under consideration.

The proposed scheme makes it possible to achieve correctness in probabilistic and statistical modeling, as well as to best take into account the opinion of the DM about the type of LRO to which the elements of a variety of states of the economic environment are subject, including for cases where the use of traditional point estimates is impossible because the most characteristic estimate of the probability distribution should differ from the uniform law.

The properties of the FS completely coincide with the properties of the sequence generating it (with the exception of the normalization condition, which the sequence generating the FS does not have to satisfy). When choosing an FS that has all the desired properties, we can limit ourselves to considering the set of generalized Fishburne progressions, which are FS that are arithmetic or geometric progressions. In the case of a simple LRO, we can limit ourselves to considering generalized arithmetic Fishburne progressions, and in the case of a partially enhanced LRO, generalized geometric Fishburne progressions. Finally, if the DM adheres to the Gibbs–Jaynes principle, then either the uniform law (with the validity of a simple LRO) or the generalized geometric Fishburne progression maximizing the entropy value (with the validity of the corresponding partially enhanced LRO) should be used as an estimate of the unknown probability distribution.

The cases when FS should be used that satisfy partially enhanced LRO include cases when the sample data under consideration are time series characterized by a high rate of change. The DM is obliged to adhere to such an opinion in cases where, for example, there is either a pre-crisis (crisis) situation, or a sharp growth of the economy (of the relevant sector of the economy).

In cases where, according to the DM, the probability distribution must satisfy a partially enhanced LRO, it is impossible to use traditional point estimates, because the uniform law used to calculate the values of traditional point estimates does not satisfy a partially enhanced LRO. It can be said that in these cases there is a contradiction between the traditional probabilistic and statistical tools and the peculiarities of the decision-making situation, and the correctness and appropriateness of modeling require using a strictly monotonic sequence as an estimate of the probability distribution, i.e. a FS satisfying the corresponding partially enhanced LRO, for example, the generalized geometric Fishburne progression maximizing the entropy value (if the corresponding partially enhanced LRO is fair). Although the desired statistical properties of point estimates are lost (unbiased, etc.), the use of a strictly monotonous FS (instead of a uniform law) allows us to achieve the desired levels of correctness and appropriateness of modeling, as well as the best consideration of the opinion of the DM. ■

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