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On assigning service life for technical systems under inflation

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Abstract

A technical system is used by an enterprise that is a typical market participant to perform specific work. During operation, the operating characteristics of the system deteriorate. In case of a possible failure of the system, it is decommissioned and this causes losses for the enterprise. It turns out that it is beneficial to assign a certain service life to the system after which (if no failure has occurred) it is subject to decommissioning. We are solving the problem of optimizing this assigned service life. Usually, when solving it, inflation is not taken into account, and the optimality criteria are the average costs per unit of time and other indicators that do not fully reflect the commercial interests of the enterprise owning the system. Using the principles and methods of valuation, we build a mathematical model and propose formulas that allow us, taking into account inflation, to find the optimal assigned service life of the system and at the same time estimate the market value of the work performed by the system and calculate the change in the market value of the system with age. Moreover, in this problem, the optimality criterion is the ratio of the expected discounted costs to the expected discounted volume of work performed by the system. We show that such a criterion maximizes the market value of the enterprise owning the system. We give examples of using the constructed model. The results obtained can be used both for solving other optimization problems of the reliability theory and for practical valuation of some types of machinery and equipment.

Keywords: technical system reliability, failure, assigned service life, optimization criterion, valuation theory, market value

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Introduction

The establishment of the service life of machinery and equipment has been, at least since the 20th century, a topical task for both economists and technical specialists. A review of the relevant literature would require too much space, so we do not provide it. Let us just mention that back in the 1920s and 1930s, a number of articles were published on the economic aspects of optimizing the service life of machines that have not lost their relevance today. However, in their research, the economists usually ignored the stochastic nature of the machine operation process, while technical specialists focused on the issues of their reliability and repair, ignoring the impact of physical deterioration on their performance and (sometimes) on operating costs. Economists considered the purchase and utilization of machines as an investment project and accordingly relied on the theory of evaluating the efficiency of investment projects. At the same time, technical specialists, when setting the assigned service life of technical systems, chose the optimality criterion, often arbitrarily, without proper justification (for more details, see the next section). It is significant that almost all known works on this problem did not take into account the influence of inflation. Meanwhile, in the course of practical valuation of real assets (buildings, construction facilities, machinery and equipment), appraisers take into account not only inflation, but also physical deterioration and reliability of the assets being evaluated, and the appropriate valuation standards include the general principles of such assessment.

In this regard, it seems essential to combine technical and economic aspects within a single optimization model, taking into account both reliability and physical deterioration of technical systems, and relying (unlike existing economic and mathematical models) on the theory of valuation. So, the purpose of this article is to develop a model regarding the tasks of assigning service life to technical systems that are subject to stochastic failures.

1. Statement of the problem and main concepts

The objects of the study in this article are technical systems, usually machines and equipment that:

- are used by enterprises participating in the market;
- are subject to stochastic failures;
- are unrepairable, i.e. are not subject to major or medium repairs (in the case of failure, such systems are decommissioned and disposed of).

The subjects of our study are assigned service lives and market values of technical systems of different ages. We present the information related with valuation of technical systems and their reliability in accordance with valuation standards [1] and the book [2]. The main operational characteristics of a technical system (depending on its technical conditions), are:

- operational productivity (the volume of work performed by the system in a small unit of time);
- operating costs rate (operating costs related with use of the system for its intended purpose for a small unit of time);
- hazard rate (probability of failure of the system within a small unit of time).

The reliability theory considers the task of establishing the assigned service life of an unrepairable technical system. We will deal with the justification of optimality criterion of such service life. For this reason, we limit our considerations to the following situation, which is quite simple.

An enterprise acquires on the market and uses an unrepairable technical system of a certain type (for example, a machine of a certain model). During the operation, the system is degraded and its operational characteristics deteriorate, despite the ongoing maintenance (we include the costs of such operations into overall operating costs). This provision needs some comments.

Many specialists, as in the times of the "plan oriented economy," believe that such characteristics of machines and equipment as their productivity and annual operating costs must remain unchanged throughout the entire service life at the level provided for by the project. The permanent performance of the technical system is also accepted in existing publications on the problems of optimizing the assigned service life and the timing of scheduled repairs of the systems. However, an analysis of the operation of machines and equipment used for various purposes shows that with increasing age their characteristics tend to deteriorate, and overhauls compensate such deterioration only partially (for more details, see [3]). Researchers often try to explain this point by saying that only the cost and duration of maintenance grow with increasing age. Nevertheless, the data available indicate that for construction machines their productivity per hour decreases with age, fuel costs per 1 km of vehicle mileage increase and duration of the processing cycle of machine tools increases.

For this reason, it seems unreasonable to consider the operational characteristics of technical systems as permanent over time. Further, we assume that the technical condition of the system is determined by its age (operating time), so its operational characteristics may be presented as certain functions of age.

Since technical systems are freely traded on the open market and have some usefulness for enterprises participating in the market, they become objects of valuation. There are many bases of value, but the main one is the market value. We will not give a detailed definition of this concept, referring to the valuation standards [1]. However, it should be noted that the market value of an object of valuation as of a certain date (valuation date) reflects the price of this object in a transaction (real or hypothetical) made on this date between independent and economically rational typical market participants. At the same time, the market value of the object reflects both its usefulness for typical market participants and the contribution of the object to the market value of an enterprise (a typical market participant) that possesses this object.

For valuation of objects, three approaches are used (individually or in various combinations):

- using a *comparative (market) approach*, the value of an object is determined relying on the prices of transactions with similar objects, adjusted for differences in the characteristics of objects, date and conditions of the transaction;
- with the *cost approach*, the value of the object is estimated by the costs incurred during creating or receiving this object;
- with the *income approach*, the value of the object is found, taking into account the flow of benefits that the owner will receive from using the object.

Since a technical system is a plant and equipment, its market value may differ depending on whether it is evaluated "in place" or "for removal." New systems are usually sold by their manufacturers or dealers in the primary market and are similar to each other. While the price spread is small, the market value of a new object "for moving to the place of operation" may be determined using a comparative approach, which is not difficult in practice. The market value of the same technical system "at the place of its operation" is higher, since it includes additional costs for the transportation of the purchased object and its installation. We consider the market value of a new object "in place" equal to a known value K.

Used technical systems are sold on the secondary market. Such objects do not have exact analogues. Buyers do not have the opportunity to assess their technical condition, and sellers do not inform buyers (or even do not know) the history of their operation. As a result, the prices of machines or equipment of the same type and the same age have a very large dispersion, so that valuation of used objects presents significant difficulties for appraisers.

As an analogue of a used technical system, it is reasonable to consider a similar new system (we will call it a "new analogue"). The market value of a new analogue of the system is called the reproduction value of the object being valued. A decrease in the market value of a used system compared with its reproduction value is called depreciation of the appraised object, and the ratio of these values, expressed in fractions of a unit or as a percentage, is the coefficient/percentage of goodness or relative value (Percent Good Factor, PGF). In the valuation related literature, there are a number of formulas and tables describing (not always correctly) the dependence of the PGF on the age of the system [3].

A system that can no longer be used for its intended purpose or is ineffective is to be decommissioned and disposed of. The market value of such a system is called scrap value. This value is usually determined relying on the information about prices of machines sold "for scrap" or calculated as the cost of separate elements of the machine that are suitable for further use (including scrap metal), less costs of dismantling and delivery of such elements to the place of their further use. The scrap value of machinery and equipment is small in comparison with their reproduction value – the ratio of these values (relative scrap value) for machinery and equipment is usually from 0.03 to 0.09.

Methods of valuation of machinery and equipment are described in small sections of valuation standards and textbooks. As a rule, these issued take into account neither the deterioration of the operational characteristics of systems with increasing age, nor the probabilistic nature of their service life. The exception is the methods used in national accounting systems [4], but they are focused on assessing large groups of assets and inadequately take into account the degradation of machinery and equipment [5]. At the same time, it is possible to take into account these factors adequately relying on basic principles of valuation and valuation practice by using models and methods of the reliability theory.

One of the main characteristics of technical systems' reliability is the hazard rate. We will consider it as dependent on age of the technical system (t) and denote it by $p(t)$. This value has a simple "physical meaning": if the system has worked without failure for t years, then the probability of its failure in the time interval $(t,\ t + dt)$ is $p(t)dt$. In this case, the (random) moment of failure has a distribution with the density $p(t)e^{-P(t)}$, where $P(t)=\int_0^t p(x)dx$. As is known (see, for example, [2, 6]), the probability of failure–free operation of a technical system during the period t is $e^{-P(t)}$, and the average uptime (average time to failure), unless it is specifically limited, is – $\int_0^\infty e^{-P(t)}dt$.

It is essential that the failure of a technical system leads to the failure of not only the system itself, but also of other assets of the enterprise. At the same time, the damage to the enterprise is, generally speaking, stochastic. Methods for determining such damages (losses) are described in the relevant literature, for example [7]. We will assume that the average value of the damage is known, and its value takes into account the cost of the elements of the failed system and other assets of the enterprise that are suitable for further use.

In order to reduce losses because of the failure of the system, the system is assigned a certain service life (operating time) S, upon reaching which the object must be decommissioned regardless of its technical conditions. We want to find the optimal value of the period S. Selecting S also determines the average remaining service life (average uptime) of objects of different ages.

Let us consider a technical system that has survived to the age of t. Let $T(t)$ be the average remaining life of its operating. To find it, we note that the remaining service life of the object will be $x-t$ if it fails at the age of $x < S$, and will be equal to $S-t$ if no failure occurs within the designated period. Considering that the probability of failure-free operation of an object that has survived to the age of t, during the entire designated period is equal to $e^{P(S)-P(t)}$, and failure during the time interval $(t, t + dt)$, $t < S$, is possible with probability $p(t)e^{P(S)-P(t)}dt$, we get:

$$T(t)=\int_t^S (x-t)p(x)e^{P(t)-P(x)}dx+(S-t)e^{P(t)-P(S)}=$$
$$=\int_t^S e^{P(t)-P(x)}dx. \qquad (1)$$

For many types of technical systems, it is assumed that the uptime has the Weibull distribution, in which the hazard rate $p(t)$ increases proportionally to some degree of age. The values of the degree index for various types of objects were calculated by a number of authors, including for the purposes of national accounting, and the recommended values for a number of engineering facilities are given in [6]. It should be noted that for some types of machines (freezers, refrigerators, vacuum cleaners, microwave ovens, video recorders, washing machines, electric heating appliances, small cars, equipment for car repair and maintenance, railway wagons), this indicator turns out to be close to one [8–10]. Later, as an example, we will consider exactly such objects. Their uptime has a Rayleigh distribution, $p(t)=t/\omega^2$, where ω is the scale parameter. In this case, the average remaining service life of an object of age t will be

$$T(t)=e^{t^2/2\omega^2}\int_t^S e^{-x^2/2\omega^2}dx=$$
$$=\omega\sqrt{2\pi}\left[\Phi\left(\frac{S}{\omega}\right)-\Phi\left(\frac{t}{\omega}\right)\right]e^{t^2/2\omega^2}.$$

At $t = 0$ the formula for the average full service life of the technical system is:

$$T_m=T(0)=\omega\sqrt{2\pi}\left[\Phi\left(\frac{S}{\omega}\right)-\frac{1}{2}\right]. \qquad (2)$$

At $S=\infty$, when the service life is not assigned, the average uptime of the system will be equal to $\omega\sqrt{2\pi}$.

In the reliability theory, various criteria are usually used to solve optimization problems. The examples are: the average number or cost of repairs over the service life, average repair costs per unit of time [11], average costs per unit of time for the inter-repair cycle [12], the ratio of average lifetime costs to average service life [13–15], total discounted costs or an annuity equivalent to them [16, 17]. These criteria have a number of common disadvantages.

1. Selection of the criterion is of a formal mathematical nature and is focused on the use of the measured characteristics of the system (primarily, related with cost and time). The interests of a particular business using a technical system are not taken into account [12].

2. Comparing options of applying technical systems using costs is correct only if they give identical results [18]. However, if the performance of a system changes with age, then options with various service lives assigned will have differences in the value and variances of results obtained over time, i.e. volumes of work performed.

3. Calculating indicators such as total or average lifetime costs, does not take into account the impact of inflation. This could be justified if we were talking about systems with a short (up to 1–2 years) service life. However, in con-

struction, engineering and transport industries these terms exceed 5–15 years, and in such cases, consideration of inflation is essential.

Taking into account the points mentioned above, we consider that when solving the task it is more appropriate to apply general principles of valuation focused on maximization of the market value of enterprises.

At the same time, unlike works on reliability theory, it turns out to be possible to take into account not only inflation, but also the impact of technical systems' degradation on their performance[1] and operating costs, as well as the scrap value of the systems. Taking into account inflation matters is related with special considerations which the next section will be devoted to.

2. Group inflation

Selecting economically rational solutions for managing technical systems under conditions of inflation is associated with significant difficulties. The fact is that this requires a forecast of the economic characteristics of the system for its entire service life, which implies, at a minimum, forecasting prices for products produced by the systems (goods, works or services), as well as for various resources consumed. Unfortunately, specialists in the management of technical systems are not able to develop such forecasts. That is why in mathematical models of optimization of technical solutions focused on practical application, inflation is usually not taken into account. This also applies to the task of assigning the service life for a technical system. Meanwhile, it is possible to find out how inflation affects the solution of this problem, relying on the theory of valuation and the practice of valuation activities. It is proposed to take into account only the most important and measurable characteristics of inflation, neglecting all the others.

Analysis of the prices in primary and secondary markets, as well as the experience of evaluating used machines and equipment show that the prices of used machines usually change synchronously with the prices of similar new ones. This is due to the fact that the "economically rational" buyer of a used machine (on the secondary market) compares the planned purchase with the alternative of purchasing its new analogue on the primary market.

In this regard, it seems natural to assume that in conditions when the market values of new technical systems of some kind are growing, the market value of used systems of this type are growing in the same proportion. This phenomenon is called group inflation [3]. However, it is not easy to give a strict definition of group inflation, since all used systems of the same type, unlike new ones, have no analogues, are in different technical condition, their quantity and composition on the secondary market are constantly changing. Therefore, the concept of growth of systems values on the secondary market becomes uncertain.

Nevertheless, the essence of group inflation can be explained by a conditional example relating to a group of machines of the same model, the technical condition of which changes over time. Let us imagine a market in which at some time T_1 there is a set of C_1 of M new and used machines of this group, which are in different states, and the costs of all these machines are known. Now let us assume that at a later moment T_2, a set of C_2 of M machines of the group also appeared on the market, and each i-th of them is in exactly the same technical condition as the i-th machine from the set of C_1. In other words, in this situation, all the

[1] In the works on the problem of optimizing the assigned service life and the timing of scheduled repairs of technical systems, their performance is usually considered as unchanged.

machines of the group available on the market at the time T_1, as if "moved" to a later date without changing their technical conditions. At the same time, due to inflation, the market values of all the machines will change. However, under group inflation, they will change proportionally, and it turns out that the ratio of the value of a used machine to the cost of the same new one (the PGF) will depend only on the condition of this machine, but not on whether it is valued at time T_1 or at time T_2. This statement will be the basis for the definition of the concept of group inflation. We will say that for a certain type of a technical system, group inflation takes place in a certain time interval if the PGFs of these machines depend only on the condition of the systems, but not on what date (in the specified time interval) their value was estimated. The rate of group inflation is defined as the growth rate of the market value of new machines. Therefore, for the analysis and short-term forecast of this rate, it is sufficient to use open and accessible information about the prices of manufacturers or dealers in the period close to the valuation date.

Group inflation assumption significantly simplifies the solution of the problem of optimizing the assigned service life of a technical system, as we will see in the next section. However, the validity of this assumption is also confirmed by other arguments.

Usually, the technical condition of used machines sold is unknown to appraisers[2]; they only know their age. Therefore, they have to characterize the condition of the machine by its age (as in this article). Then, for an approximate calculation of PGF, they form a sample of the market prices of machines of different ages in a certain base period (for example, in the current year) and build a regression dependence $F(t)$ of machines' prices on their age (t). At the same time, the value of $F(t)$ will reflect on this basis the average market value of machines of age t that were (or could be[3]) presented on the market in the base period, and $F(0)$ – the market value of new machine in this period. The goodness factors are found by the formula: $k(t) = F(t)/F(0)$. In the absence of group inflation, similar dependencies constructed for machines of the same model according to data from different years could differ significantly (this would mean, for example, that under the influence of inflation, the prices of older machines decrease or grow more slowly than the prices of younger ones). However, this phenomenon is usually not observed.

On the contrary, under conditions of group inflation, the function $k(t)$ does not depend on which period the prices of machines belong to. Particularly, Russian appraisers usually use the dependence of PGF of machines on the age, built on the market data of previous years by other authors. By doing so, they are essentially also assuming group inflation assumption. Approximately the same situation appeared in some US states where machinery and equipment are subject to taxation. There, regression dependences of PGF on age are built annually, though for quite wide groups of machines (for example, agricultural or construction machinery and equipment). An analysis of relevant publications (for example, [19–21]) shows that these dependencies change slightly from year to year, which also indicates group inflation.

[2] It would seem that quantitative assessments of the technical condition of machines and equipment are provided by automated diagnostic systems. However, these systems do not assess either the hazard rate of the machine or other important characteristics for market participants (for example, the condition of the frame of a truck or the body of a passenger car significantly affecting their market value).

[3] This stipulation is related with the fact that the constructed dependence allows us to calculate $F(t)$ for machines of any age t, whereas in the base period machines of some ages were not available on the market.

It is important to note that the assumption of group inflation allows us, when constructing the dependence $k(t)$, to include in the sample the prices of machines that have developed in different years (while simultaneously calculating the market value of new machines in these years). This makes it possible to significantly increase the sample size and improve the accuracy in calculating dependence $k(t)$.

Of course, in other situations, group inflation can be understood in a different way, using the operating time or other objectively measurable characteristics instead of age. In such cases, the following model will need to be adjusted accordingly.

3. Optimization model

To optimize the assigned service life for a technical system belonging to some type, it is necessary to set a certain optimality criterion. For this purpose, we will solve another problem that, at first glance, is quite far from the theory of reliability: we will evaluate the market values of technical systems of different ages "in place." As the valuation date, we will take the moment of making a decision on the assignment of the service life for a system. In addition, we assume that for an object of this type for a short period (close to the valuation date) there is a group inflation with a known rate i.

We will also consider as known the following characteristics of the system: market value of a new object K, the scrap value U, the average (expected) damage caused from the failure of the system L, hazard rate $p(t)$ of an operable system of age t, its operational performance $Q(t)$ and the intensity $C(t)$ of its operating costs[4].

The assigned service life of the system to be optimized is denoted by S, while market value of an operable system of age t "on the spot" is denoted by $V(t)$.

Note that the work performed by the system has some utility for market participants and is measured in certain physical units (cubic meters of displaced soil, decaliters of spilled liquid, the number of conventional cans of canned food, etc.). If so, then, according to the valuation standards [1], it also has its market value. Another thing is that the owners of machines usually do not know it, and professional appraisers rarely evaluate the market value of work (except, perhaps, construction and repair works). The unknown market value of a unit of work performed by the system at the valuation date is denoted by B.

In general, the benefits from the use of an object for a period are defined as market value of works performed by the system during this period, minus costs incurred. Depending on the method of measuring values and costs, the following types of benefits are distinguished:

- if values and costs are measured in prices of the corresponding period (i.e. including inflation), the benefits are called *nominal*, while if prices of a certain fixed date are used – they are *real*;
- if income tax is taken into account as a part of costs, the benefits are called *after-tax*, otherwise they are *pre-tax*.

In this paper, the benefits of using the technical system are considered as nominal and pre-tax. In this case, the intensity of benefits from using an operable system of age t (i.e. the volume of benefits the system brings in a small unit of time) will be $BQ(t) - C(t)$ at the valuation date.

[4] Strictly speaking, the productivity and operating costs of a system depend on variances in demand for the products of enterprises using the system, and, consequently, on the need for the work performed by the system. In our model, $Q(t)$ and $C(t)$ are average (for typical enterprises – owners of the systems) values of appropriate characteristics relevant to the average mode of applying operable systems of age t.

To evaluate a technical system, we will use the anticipation of benefits principle, which underlies the income approach to property valuation and is mentioned (but not disclosed) in the valuation standards. We will give its most detailed formulation following [3].

Market value of an object at the valuation date is equal to the sum of the discounted by this data benefits from its use anticipated in the subsequent forecast period (including market value of the object at the end of the period), if the object is used most effectively, and not less than it otherwise.

A number of comments should be made to this formulation.

1. The duration of the forecast period can be chosen arbitrarily, since market value of an object does not depend on this choice.

2. Inclusion of the object value at the end of the period in the total benefits for the forecast period can also be treated as the benefit from the sale of the object at market value. Thus, the sale of an object is considered as one of the ways to use it.

3. In the conditions of stochasticity, the term "expected" is treated as an expectation (in [1] – "probability-weighted"), and the risks associated with the uncertainty of benefits are not taken into account in the discount rate (we will call such a rate "risk-free").

4. The type of discount rate is determined by the type of benefits: nominal (real) benefits should be discounted at the nominal (real) rate, and after-tax (pre-tax) benefits – at the after- (pre-) tax rate[5]. Therefore, in this article, when evaluating a technical system, we use the nominal pre-tax risk-free discount rate, denoted by *r*.

5. Valuation standards [1] require that, when assessing any type of object value, you specify the premise regarding the way of its use. Here and further, the way of using the object is assumed to be the most effective (in valuation standards terminology – "highest and best"), so that the appropriate premise is omitted.

Turning to the definition of unknown values $V(t)$, first we note that market value of a new technical system is a known value $V(0) = K$. In addition, a system reached the designated service life S is disposed of, so that its market value is $V(S) = U$.

Let us consider a technical system that has reached the age of $t < S$. To evaluate its market value $V(t)$ at the valuation date, we apply the anticipation of benefits principle regarding this system, choosing a forecast period of infinitesimal duration dt. At the same time, we take into account that in our model all possible ways of using a system differ only in the assigned service life S.

If a failure occurs during dt, which is possible with a probability of $p(t)dt$, then the average damage for the enterprise is L, and the system will be disposed of. In the opposite case, the system will bring benefits $[BQ(t) - C(t)]dt$, and at the end of the period its age will be equal to $t + dt$. A technical system of this age at the assessment date has market value $V(t + dt)$. However, under group inflation at a rate of i during dt, it will grow by $1 + i\,dt$ times and become equal to $(1 + i\,dt)\,V(t + dt)$. Considering the probabilities of both cases, we get:

$$\begin{aligned} V(t) \geq\ & p(t)dt \cdot (-L) + \\ & + \left[1 - p(t)dt\right]\left\{\left[BQ(t) - C(t)\right]dt + \right. \\ & \left. + (1 - rdt)(1 + idt)V(t + dt)\right\}, \end{aligned} \quad (3)$$

moreover, equality is achieved here with the most efficient use of the system, i.e. with the optimal value of S.

Let us now introduce the "real"[6] discount rate into consideration:

[5] Appraisers determine discount rates based on data on nominal pre-tax yields of financial instruments observed in the market.

[6] The term "real" is taken in quotation marks, since in formula (4) the nominal rate is reduced by the rate of group inflation, and not by the rate of general inflation in the country, as is required when calculating the proper real rate.

$$\rho = r - i. \tag{4}$$

Using it, after simple transformations of inequality (3) we find:

$$BQ(t) - C(t) - \rho U - (L+U)\,p(t) \le$$
$$\le \left[\rho + p(t)\right]\left[V(t) - U\right] - \left[V(t) - U\right]'.$$

Let us multiply this inequality by $e^{-\rho t - P(t)}$ and integrate by t from $t = s$ to $t = S$. Considering that $V(S) = U$, we get:

$$\int_s^S \left[BQ(t) - C(t) - \rho U - (L+U)\,p(t)\right] e^{-\rho t - P(t)} ds \le$$
$$\le \left[V(s) - U\right] e^{-\rho s - P(s)}, \tag{5}$$

or:

$$V(s) \ge U + BQ_\Sigma(s,S) - C_\Sigma(s,S), \tag{6}$$

where:

$$\begin{cases} Q_\Sigma(s,S) = \int_s^S Q(t)\, e^{-\rho(t-s) - P(t) + P(s)} dt, \\ C_\Sigma(s,S) = \int_s^S \left[C(t) + \rho U + (L+U)\cdot \right. \\ \qquad \left. \cdot p(t)\right] e^{-\rho(t-s) - P(t) + P(s)} dt. \end{cases} \tag{7}$$

Substituting $s = 0$, $V(0) = K$ into formula (6) and expressing B from it, we get:

$$B \le \frac{K - U + C_\Sigma(0,S)}{Q_\Sigma(0,S)} = Z. \tag{8}$$

In this case, the equalities in (5), (6) and (8) are achieved with optimal S.

It follows that with the optimal assigned service life, the minimum of the Z index, which has the form of a fraction, is achieved. Let us find out the economic meaning of its numerator and denominator using formulas (7).

At first glance, it is obvious. The denominator of the fraction is the expected discounted amount of work performed by the system during the entire assigned service life, and the numerator is the expected discounted costs of its acquisition and use (less disposal value of the system)[7], including damage from failures. So, the fraction Z represents the expected specific (per unit of work) discounted costs (ESDC).

In fact, this term is not quite correct, since the values $Q(t)$, $C(t)$, $p(t)$ included in formulas (7) reflect not the dynamics of the characteristics of an operable system over its service life, but the characteristics of operable systems of different ages at the assessment date. Only when all prices in the country grow at the same rate i throughout the life of the system, the numerator and denominator of the fraction Z can be interpreted as the expected discounted costs and results associated with the acquisition and use of the system until the end of its service life. Nevertheless, the introduced term and its abbreviation ESDC seem convenient and visual, and we will use them.

As can be seen from formula (8), the optimal assigned service life should correspond to the minimum value of the ESDC. Moreover, this value will be equal to the market value of the unit of work performed by the object B. Such a method of evaluating works' market value is consistent with a cost approach to valuation, although appraisers do not use it in this form.

We also see that the application of the ESDC criterion orients the enterprise to the most efficient use of the technical system and maximizes its market value[8]. A similar criterion of discounted costs per unit for determining the service life of machines in a deterministic situation was previously justified using optimization models (see, for example, [18, 19]). Criteria similar in content were also used in solving

[7] Since the discounted performance and costs relate to an operable (not failed) technical system, for discounting we apply a "real" rate which takes into account the risk of failure depending on the age of the object $\rho + p(s)$.

[8] The market value of the enterprise, assuming that it assigns a different service life to the object, will decrease.

some optimization problems of the reliability theory (see, for example, [23, 24]); however, the productivity and intensity of operating costs of the system were assumed to be constant, and its disposal value was not taken into account.) In contrast to the criterion of the ratio of average costs to average service life, often used in works on the theory of reliability, the ESDC takes into account the different timing of costs and results of the system, but does not consider the time to eliminate the consequences of failures.

The calculation of the optimal value of S and the cost of a unit of work B can be simplified by using the inequality (5). Since the integrand in (5) decreases with the growth of t, it is not difficult to make sure that the maximum in the left part of (5) is reached when S is the unique root of the equation: $BQ(S) - C(S) - \rho U - (L + U)p(S) = 0$. In this case, inequality (8) becomes equality, and S and B will be the solution of a system of equations:

$$B = \frac{K - U + C_\Sigma(S)}{Q_\Sigma(S)} = \frac{C(S) + \rho U + (L + U)p(S)}{Q(S)}. \qquad (9)$$

The dependence of the cost of the technical system on its age now follows from inequality (6), which at optimal S becomes equality:

$$V(s) = U + BQ_\Sigma(s, S) - C_\Sigma(s, S). \qquad (10)$$

Substituting in (10) B from the first equality (9), we get:

$$V(s) = U + (K - U)\frac{Q_\Sigma(s, S)}{Q_\Sigma(S)} + \left[C_\Sigma(S)\frac{Q_\Sigma(s, S)}{Q_\Sigma(S)} - C_\Sigma(s, S)\right]. \qquad (11)$$

This equality, in fact, is one of the modifications of the well-known Lvov formula [25], which was used in the USSR to evaluate the efficiency of new equipment and set prices for new technology, and is currently used in the machinery and equipment valuation (for the history of the Lvov formula and its modifications, see [3, 26, 27]). Unlike other modifications of the Lvov formula, (11) takes into account the scrap value of the system, as well as the risk of its failure and is applied to the used object valuation.

The characteristics of machines and equipment used in their valuation include average full and residual service life. They can be calculated for the systems under consideration. It would seem that for these purposes, formulas (1) and (2) can be used, linking the specified terms with the assigned service life (S). However, this would not be quite correct, since when deriving formula (1), it was assumed that until the system is disposed of, its assigned service life does not change. Such an assumption would be justified if, starting from the valuation date, group inflation with a constant rate would take place for the system. However, the service life of a system can be tens of years, while the rate of price growth for machines can fluctuate considerably. In this case, the optimal values of the assigned service life of the system will also change. Under such conditions, the values of $T(t)$ and T_m calculated by formulas (1) and (2) can no longer be called the average residual and full service life of the object. Rather, they reflect such terms calculated under the assumption of the group nature and the constant rate of inflation throughout the designated service life of the system. However, similar stipulation should be made regarding other technical and economic standards (for example, standards for frequency of repairs or safety margin factors), because they all were developed for specific economic conditions and therefore will be economically justified until these conditions change significantly.

4. Experimental calculations

According to the model constructed, experimental calculations were carried out in several variants. Time was measured in years and fractions of a year. In all variants, $V(0) = K = 100$, $Q(0) = 1$ were accepted. The market values of technical systems of different ages at the same time numerically coincide with their PGF. It was assumed that the dependencies $C(t)$ and $Q(t)$ are linear, and the uptime of the systems has a Rayleigh distribution:

$$Q(t) = 1 - \alpha t,\ \ C(t) = C_0(1 + \beta t),\ \ p(t) = t/\omega^2,$$

where α and β – (basic) rates of productivity decline and growth of operating costs, respectively, 1/year;
ω – parameter od the Rayleigh distribution, years.

The optimal assigned service life of the system (S) and market values of units of work performed by the system on the valuation date (B) were found by numerical solution of the system of equations (9).

The basic variant was adopted, with the following values of parameters: $U = 7$; $C_0 = 100$; $L = 100$; $\rho = 0.1$; $\alpha = 0.01$; $\beta = 0.02$; $\omega = 8$.

By varying the parameters of this variant, it is possible to identify their influence on the optimal value S of the assigned service life of the system (*Figs. 1–4*).

Note also that the optimal S decreases slightly with the growth of the scrap value U and increases with the increase in the discount rate (especially strongly – with small damage values L).

The dependences of the average service life calculated by formula (2) on the intensity of operating costs at the beginning of operation (C_0) and ω at different damage values are shown in *Figs. 5–6.*

As one can see, new systems that require relatively large (compared to their value) operating costs have shorter average service lives. With a decrease in the failure rate (an increase in ω), the average service life of a system increases, but only up to a certain extent (the "economic" service life of similar objects that are not subject to failures).

The average market values of used objects as a function of age can be calculated using the formula (10). However, it is difficult to compare the appropriate graphs, since the average and assigned

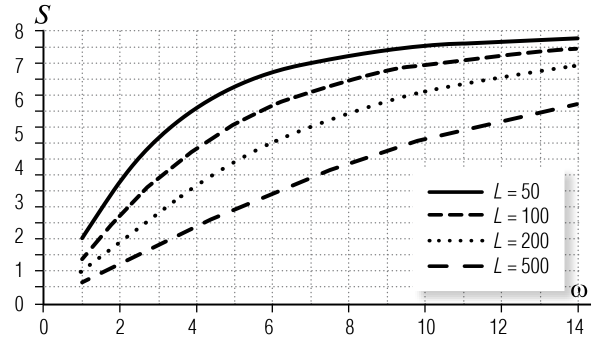

Fig. 1. Influence of ω on the optimal assigned service life under different L.

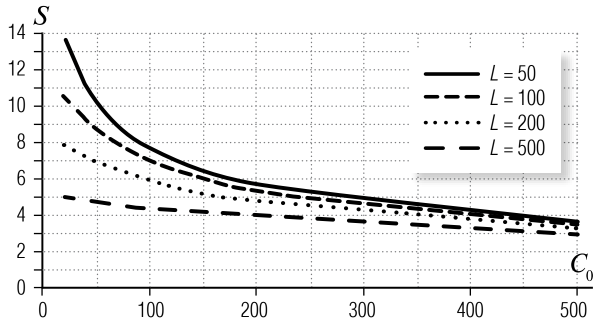

Fig. 2. Influence C_0 on the optimal assigned service life under different L.

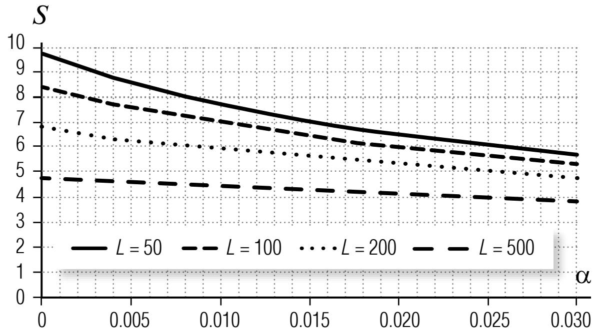

Fig. 3. Influence α on the optimal assigned service life under different L.

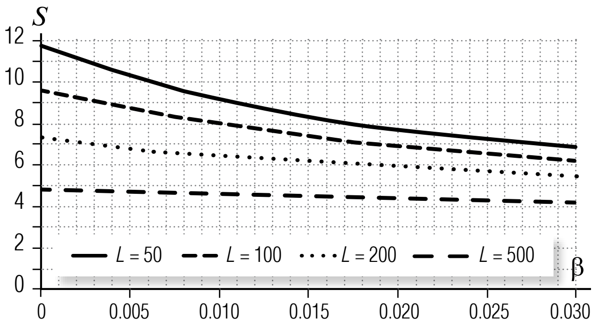

Fig. 4. Influence β on the optimal assigned service life under different L.

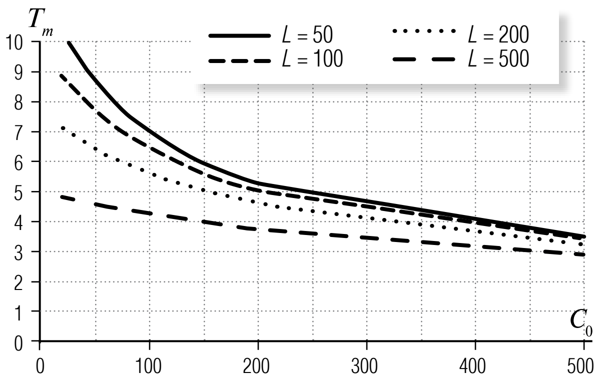

Fig. 5. Influence C_0 on the average service life of the system under different L.

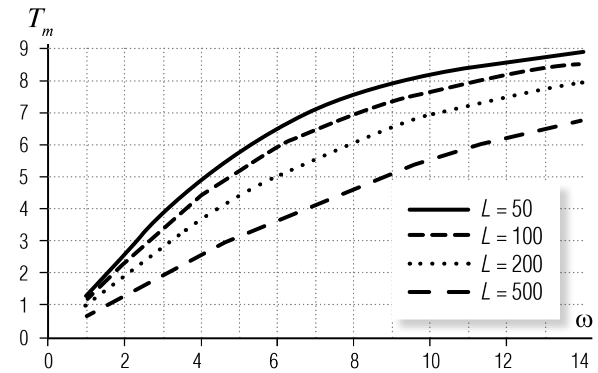

Fig. 6. Influence ω on the average service life of the system under different L.

service life for different variants vary significantly. However, the situation changes if, on the basis of these calculations, we plot the dependence of the average PGF on the relative age (τ) – the ratio of the age of the object (t) to the average period of their service (T_m).

Four variants of such dependencies are presented in *Fig. 7.* In all these variants, it was assumed that $K = 100$; $U = 7$; $Q_0 = 1$; $\rho = 0.1$; $\alpha = \beta = 0.01$. The values of other parameters of systems, as well as assigned and average service lives for these variants are summarized in *Table 1.*

Note that, despite significant differences in the main parameters of the system, the appropriate graphs turn out to be quite close. At the same time, they give lower PGF than when using the hyperbolic model adopted in the Russian system of national accounts for valuation of machinery and equipment.

Table 1.

Main parameters of technical systems by variants

Variant	1	2	3	4
C_0	20	100	40	300
L	100	200	200	500
ω	10	10	5	5
S	13.36	7.44	4.94	2.78
T_m	10.26	6.80	4.24	2.64

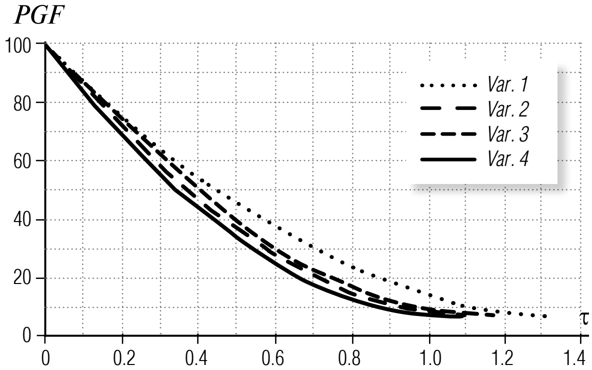

Fig. 7. Dependence of average Percent Good Factor (*PGF*) of a technical system on the relative age (τ) by variants.

Conclusion

The traditional approach to setting the assigned service life of technical systems does not fully meet the interests of market participants. To solve this problem, it is proposed to focus on maximizing the market value of the enterprise – owner of the object and to rely on the theory of valuation. It turns out that this approach makes it possible to consider inflation quite simply. In addition, the assigned service life of the object should provide the minimum expected unit discounted costs – the ratio of the expected discounted costs of acquiring and using the system and eliminating the consequences of its failures to the expected discounted amount of work performed by the system. At the same time, the mathematical model we developed allows us to estimate the market value of the work performed by the system and establish the dependence of the market value of the object on its age. The proposed approach can also be used in a situation where the degradation of an operable object is described by random processes, as well as when solving other optimization problems of the reliability theory, for example, to optimize the schedule of overhauls. ■

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