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Entropy approach to the analysis of banks' balance sheets

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Abstract

Accounting ensures the collection and systematization of documented information about the facts of the economic life of enterprises and organizations. The information collected is systematized and formalized in various forms of reporting. One of the key forms of reporting is the balance sheet. The balance sheet is based on the principle of double entry, according to which each change in the financial resources of the organization is reflected in at least two accounts related assets and liabilities. Thus, the condition of the balance of the volumes of the generalized values of assets and liabilities is realized. The control element of the balance sheet is the equality of the values of assets and liabilities. However, this control element does not allow us to identify the systemic difference (diversity) of balance sheets with equality of distributed funds. Namely, the equality condition is integral in nature and its fulfillment is not related to the specific nature of item-by-item distributions, since, at a given size of the total cost of the balance sheet, the condition can be fulfilled by various options for the distribution of financial resources by assets and liabilities. Therefore, within the framework of this article, an attempt has been made to introduce a new control element of the balance sheet, taking into account the uneven distribution of financial resources by assets and liabilities of credit and financial organizations.

Keywords: modeling, entropy, balance sheet, assets, liabilities

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Introduction

In the practice of managing credit and financial organizations, the problem of "mortality" of banks is known. According to the data of the Central Bank of the Russian Federation for the period from 2001 to 2022, the number of operating banks decreased by more than 70% (*Fig. 1*).

As the number of active players in the banking sector of the economy decreases, it becomes an increasingly urgent task to identify negative changes in the state of each particular bank.

In order to solve this problem constructively and objectively, we will consider the bank as a managed system and the system of spending financial resources on ensuring the bank's vital activity as a manager (*Fig. 2*).

The approach to the totality of expenses as a management system is based on the simple fact that each expense has a dual nature of influence on the state of the bank. On the one hand, each expense reflects the bank's need for a specific economic resource. On the other hand, each expense makes a change in the state of the control system as a whole, since it changes the values of the cost shares for various items. This prop-

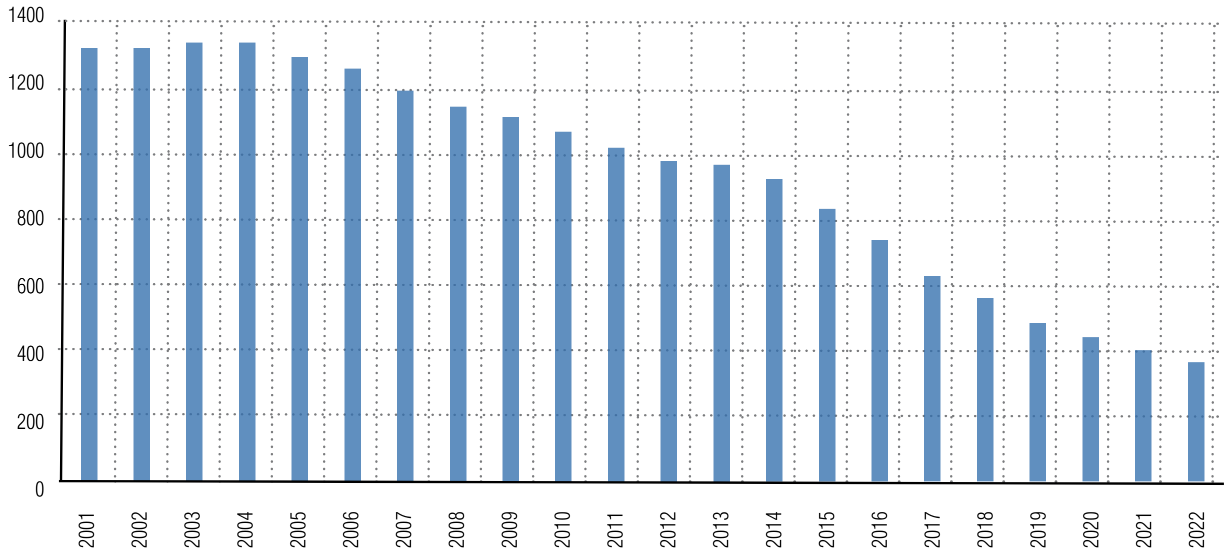

Fig. 1. The number of operating banks.

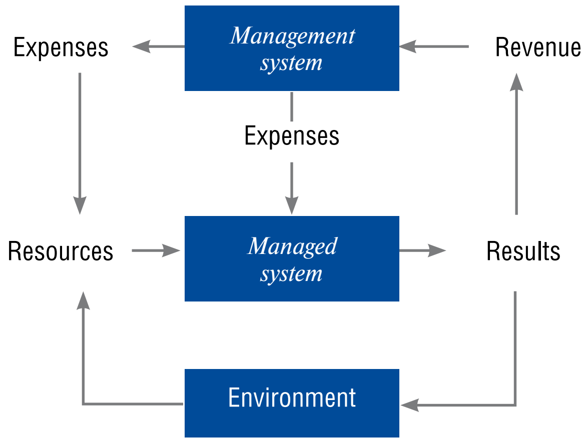

Fig. 2. The scheme of division of the control system into two parts: management and managed system.

erty of the totality of expenses allows you to manage the state of the bank as a single economic system. In the theory and practice of managing complex systems (see, for example, [1–3]) such fundamental concepts as entropy and a variety of state variants of control systems are widely used [4]. It is the variety of states of the control system that makes it possible to respond appropriately to changes in the state of the controlled system. However, in the theory and especially in the practice of balance sheet analysis, the use of the diversity property of the totality of expenses is limited by the lack of suitable methodological approaches. In this regard, the use of entropy as a measure of the diversity of the balance sheet and its application in modeling decision-making processes in the economy (see, for example, [5–14]) allow you to explore a wide variety of aspects and features of the state of the control system.

The purpose of the study is to substantiate the possibility of applying an entropy approach to assessing the diversity of assets and liabilities of the balance sheet, which may reveal the presence of an imbalance in the state of the bank.

1. The diversity of states and entropy of the management system

In order to move from "diversity" as a concept to "diversity" as a parameter of the state of the bank's management system, let us consider as a first approximation the number of options for managerial decisions on item-by-item financial expenditures. To this end, we present the totality of expenses in tabular form (*Table 1*), in the left part of which there is a list of expenditure items $\mathbf{N} = (n_1; n_2; \ldots; n_N)$, and in the right – the corresponding amounts of expenditure of financial resources $\mathbf{G} = (G_1; G_2; \ldots; G_N)$.

Table 1.

Tabular form of representation of the state of the control system

Item of expenses	The amount of funds
n_1	G_1
n_2	G_2
…	…
n_N	G_N

The tabular form (*Table 1*) allows you to enter the parameter of the diversity of the state of the control system as the number of options W, which can be implemented by a state with fixed values of the number (number) of articles N and the amount of funds S_N.

Let's explain what has been said with a simple example. Let the number of volumes of distributed funds be a vector $\mathbf{G} = (5, 20, 75)$. Then, changing the order of comparison of the elements of the numerical series $\mathbf{G}$ and the list of articles $\mathbf{N}$, we get $W = 6$, since for $N = 3$ the number of variants is equal to the number of permutations: $W = N! = 3! = 1 \cdot 2 \cdot 3 = 6$ (*Table 2*).

Table 2.

The variety of management solutions

The amount of funds		Variant "1-2-3"	Variant "1-3-2"	Variant "2-1-3"	Variant "2-3-1"	Variant "3-1-2"	Variant "3-2-1"
$G_1 = 5$	⇔	n_1	n_1	n_2	n_2	n_3	n_3
$G_2 = 20$	⇔	n_2	n_3	n_1	n_3	n_1	n_2
$G_3 = 75$	⇔	n_3	n_2	n_3	n_1	n_2	n_1

Further, it is easy to establish that any coincidence of values in the series $\mathbf{G} = (G_1, G_2, ..., G_N)$ reduces the value of the parameter W. In the limit, when the values of all components of the vector **G** are equal to each other ($G_1 = G_2 = G_3 =...= G_N$), all variants of the state of the control system are identical to each other, since the permutation of articles in this case no longer plays any role. This feature of the parameter W indicates the importance of studying precisely the unevenness of the distribution of cost values. That is, it is advisable to use a system parameter that would allow us to distinguish numerical series (vectors) **G** by the nature of the distribution of the values of their components. In fact, this will answer the question, what is the difference between numerical series with equal values of the parameters N and S_N. Such a system parameter could be the entropy of arbitrary numerical series, which can be used to quantify the diversity (distinctness) of the states of the control system. For example, what is the systemic difference between specific numerical series (vectors) $\mathbf{G'} = (3, 7, 10, 20, 60)$ and $\mathbf{G''} = (5, 10, 15, 20, 50)$, corresponding to different states of the control system? To do this, we first represent the coordinates of the vectors in the form of piecewise linear graphs constructed using a modified Lorentz diagram construction technique. Then, approximating the Lorentz diagram with a special one-parameter function, we determine the parameter of the uneven distribution α (alpha) and calculate entropy as a measure of the diversity of the state of the control system.

2. Lorentz's diagram of arbitrary numerical series

In economics, the method of visual representation of the uneven distribution of income by population groups in the form of a piecewise linear graph is well known [15]. We apply a simplified method of constructing a Lorentz diagram for an arbitrary numerical series. Let us say, for example, the original number series (vector) has five elements (components) $\mathbf{G'} = (3, 7, 10, 20, 60)$ and $\mathbf{G''} = (5, 10, 15, 20, 50)$. Calculate a number of accumulated partial sums $\{S_n\}$ using the following formulas:

$$
\begin{gathered}
S_1 = G_1,\ S_2 = G_1 + G_2 = S_1 + G_2, ..., \\
S_n = G_1 + G_2 + ... + G_n = S_{n-1} + G_n, ..., \\
S_N = G_1 + G_2 + ... + G_N = S_{N-1} + G_N.
\end{gathered} \tag{1}
$$

We get the following new series (vectors) $\mathbf{S'} = (3; 10; 20; 40; 100)$ and $\mathbf{S''} = (5; 15; 30; 50; 100)$. Next, we divide each of the accumulated sums Sn by the sum of all the numbers in the original series: $S_N = 100$. As a result, we get a series of numbers (vector) $\mathbf{Y'} = (0.03; 0.1; 0.2; 0.4; 1)$ and $\mathbf{Y''} = (0.05; 0.15; 0.3; 0.5; 1)$, which are the Y_n values of the ordinate axis of the Lorentz diagram. The values of the coordinates along the abscissa axis are calculated by the formula: $X_n = n/N$, where n is the number in order for the example under consideration and $N = 5$, that is, **X** = (0.2; 0.4; 0.6; 0.8; 1). And finally, we note on the X–Y coordinate plane the points with coordinates (X_n; Y_n) inside a square with sides equal to one. As a result, we obtain Lorentz diagrams in the form of piecewise linear graphs (*Fig. 3*), which make it possible to visually distinguish **Y″** = (0.03; 0.1; 0.2; 0.4; 1) и **Y″**= (0.05; 0.15; 0.3; 0.5; 1).

Obviously, Lorentz's diagrams allow you to visually represent the uneven distribution of the values of elements of any numerical series that differ in the number of N elements and the total sum of S_N numbers. In addition, this approach positions any particular distribution between two extreme variants (*Fig. 3*): uniform ("A" – all numbers are equal to each other) and extremely uneven ("M" – one number is significantly larger than the others). For clarity, *Fig. 3* shows two Lorentz diagrams constructed for $\mathbf{G_A}$ = (20, 20, 20, 20, 20) and $\mathbf{G_M}$ = (1, 2, 3, 9, 85).

Using the approximation of Lorentz diagrams by a family of one-parameter functions $L(x, \alpha)$ in the form

$$L(x,\alpha) = 1 - \sqrt[\alpha]{1 - x^{\alpha}}, \qquad (2)$$

we introduce the parameter α as a measure of the unevenness of the distribution of values of a particular numerical series (*Fig. 4*) [16, 17].

3. The entropy of a numerical series

In nonequilibrium statistical mechanics, knowledge of the statistical density of the distribution provides complete knowledge of the state of the system [18]. The introduced one-parameter function (2) for the approximation of Lorentz diagrams allows us to obtain an expression for the statistical probability distribution function (probability density) in the following form [16]:

$$\rho(g,\alpha) = \frac{1}{\alpha - 1} \frac{g^{\frac{2-\alpha}{\alpha-1}}}{\left(1 + g^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha+1}{\alpha}}}, \qquad (3)$$

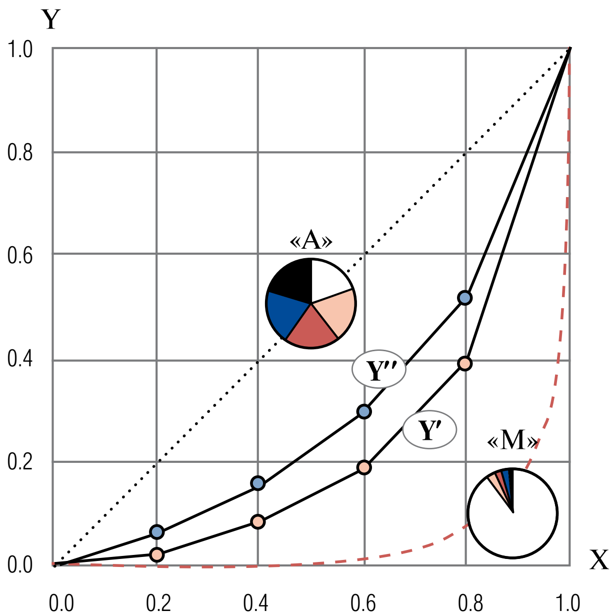

Fig. 3. Lorentz's diagram.

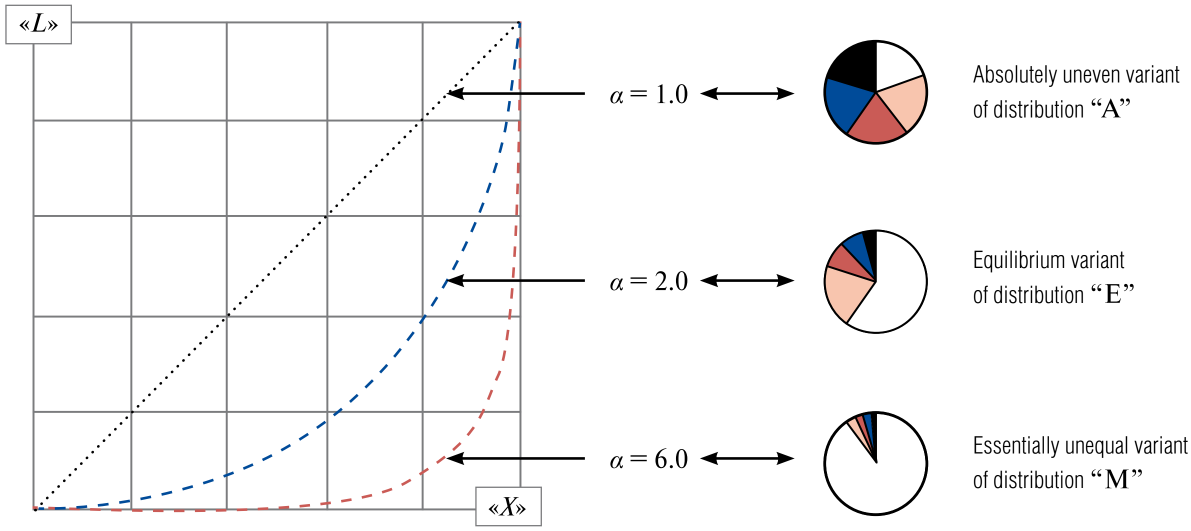

Fig. 4. The family of approximating functions $L(x, \alpha)$.

where g is the value of the flow rate G, normalized (divided) by the value of the average flow rate S_N/N.

And finally, the obtained one-parameter density of the probability distribution (3) allows us to calculate the entropy $V(\alpha)$ for an arbitrary numerical series (vector) **G** [19–21]:

$$V(\alpha) = -\int_0^\infty \rho(g,\alpha)\cdot \ln[\rho(g,\alpha)]\, dg. \qquad (4)$$

Figure 5 shows a graph normalized by the maximum value of entropy $V(\alpha)$, obtained by numerical integration of formula (4) [20]. The graph shows the entropy values for the numerical series used in the article: $\mathbf{G_A}$, $\mathbf{G_M}$, $\mathbf{G}' = (3, 7, 10, 20, 60)$, $\mathbf{G}'' = (5, 10, 15, 20, 50)$, $\mathbf{G_A} = (20, 20, 20, 20, 20)$ und $\mathbf{G_M} = (1, 2, 3, 9, 85)$.

In order to give quantitative properties to the description of our approach, it has become quite productive to use a unimodal analytical function $V(\alpha)$, approximating the graph of the entropy of the number series [22]:

$$V(\alpha) = \alpha^{-\sqrt{2}} \cdot \ln \alpha. \qquad (5)$$

As a result, having the function $V(\alpha)$, it became possible on the basis of physical entropy (4) to quantitatively formulate an entropic approach to the analysis of banks' balance sheets.

4. Entropic approach to the analysis of the balance sheet

We apply an entropic approach to the analysis of budgets of various levels [22] to analyze the equilibrium of the item-by-item distributions of assets and liabilities of the balance sheet of banks.

The concept of "balance sheet" has existed for almost six hundred years. The basis of the balance sheet is the Italian mathematician, Franciscan monk Luca Bartolomeo de Pacioli, who laid down in his work [23], published in 1494, the basic principles of accounting. He proposed the principle of double entry, according to which every change in the organization's funds is reflected in at least two accounts related to the corresponding group items of assets and liabilities.

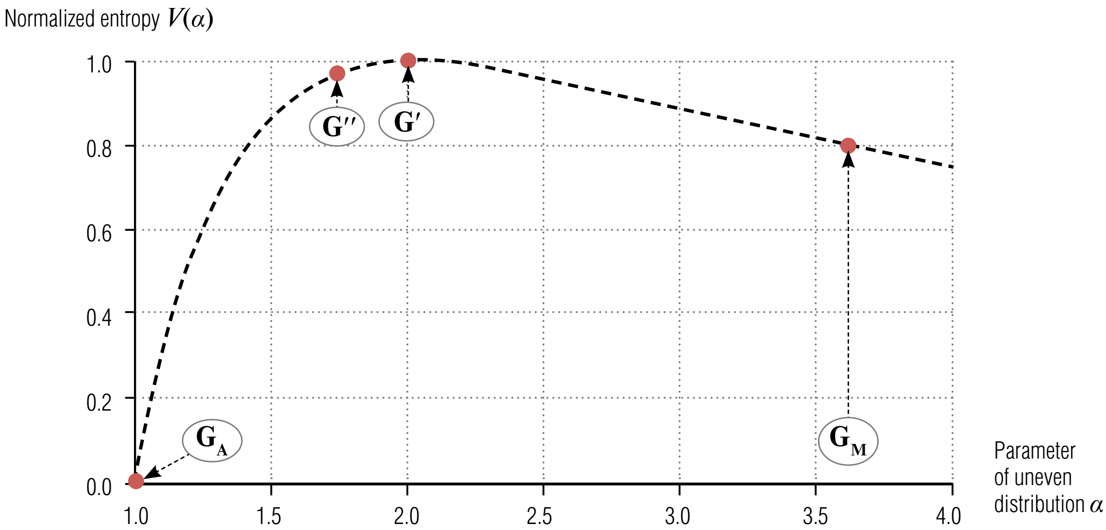

Fig. 5. The entropy of a numerical series with a different parameter of the uneven distribution.

Currently, the balance sheet is a generally accepted financial statement. The control element of the balance sheet is the equality of the value of assets and liabilities. However, the condition of equality of the value of assets and liabilities can be fulfilled by various options for the item-by-item distribution of funds. But it is the item-by-item allocation of funds that are the most important management decisions that affect the state and activities of enterprises and organizations. In this regard, a control element is needed that is directly related to the distribution of funds.

The authors tried to solve the problem by focusing on the productivity of using physical metaphors and concepts in the analysis of nonequilibrium states of macroeconomic systems [24, 25]. A convenient metaphor explaining the essence of the entropy approach to the separate assessment of the entropy of assets and the entropy of liabilities is the comparison of a credit and financial institution with a controlled section of the river. The only difference is that instead of a water flow through a controlled section of the river, we will consider the flow of entropy through a credit and financial institution. At the same time, the entropy flow control system through the organization is figuratively represented as a "gateway" in which the incoming entropy flow (V_A) is determined by the item-by-item distribution of assets (α_A), and the outgoing (V_L) is determined by the item-by-item distribution of liabilities (α_L). As a result, adjusting the distribution of assets and liabilities and calculating the corresponding entropy values (V_A) and (V_L), using formula (5), we implement entropy flow control. Entropy flow control allows us to avoid undesirable entropy "floods" (($\Delta V = V_A - V_L > 0$)) and "dehydration" ($\Delta V = V_A - V_L < 0$) in the control system. Thus, the condition of zero equality of the entropy flow $\Delta V = 0$ serves as a control element of the balance sheet, characterizing the equilibrium ("E") of the item-by-item distributions of assets and liabilities of the balance sheet.

In our opinion, the condition of equality of entropy flows is integral in nature and its fulfillment is associated with the specific nature of item-by-item distributions, since at a given size of the total cost of the balance sheet, the condition can be fulfilled by various options for the distribution of financial resources by assets and liabilities.

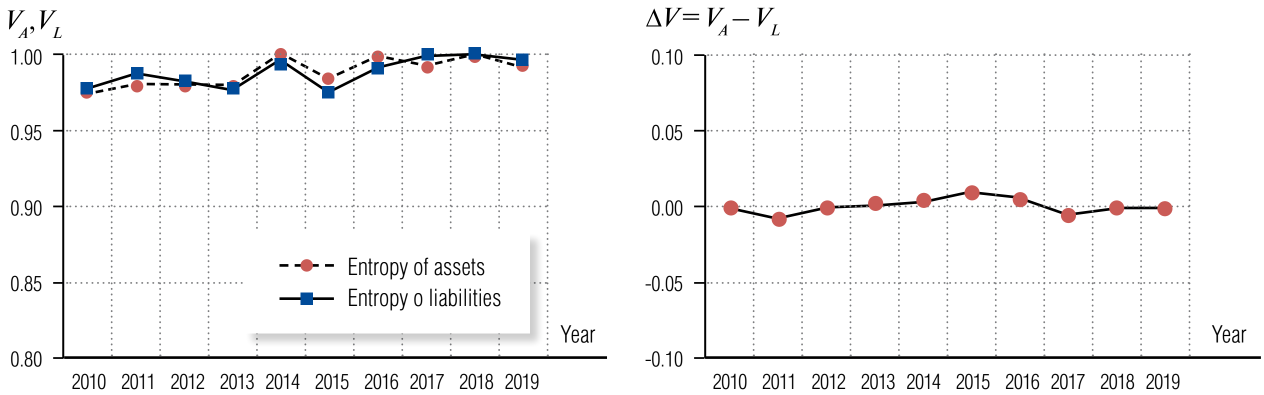

Fig. 6. Entropy of assets (V_A) and entropy of liabilities (V_L) of VTB Bank.

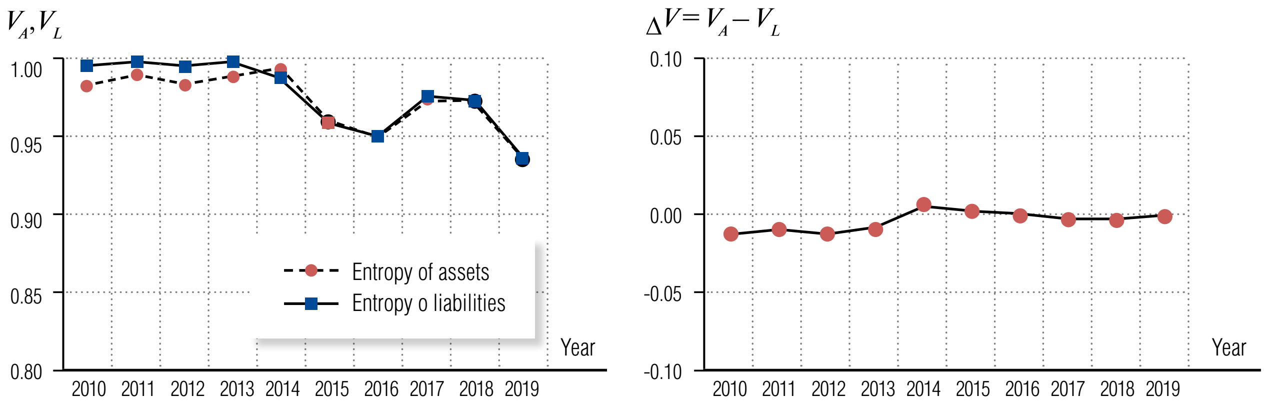

Fig. 7. Entropy of assets (V_A) and entropy of liabilities (V_L) of SberBank.

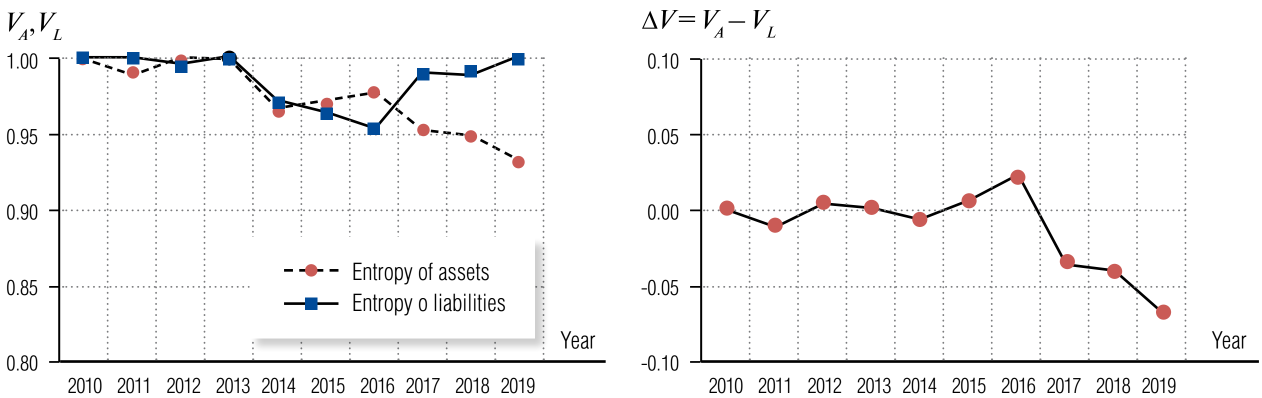

Fig. 8. Entropy of assets (V_A) and entropy of liabilities (V_L) of Promsvyazbank.

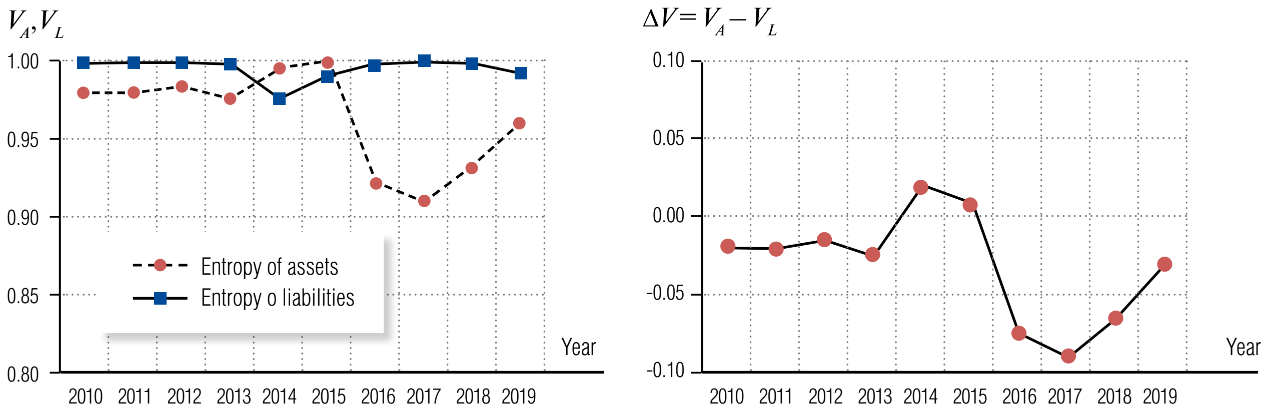

Fig. 9. Entropy of assets (V_A) and entropy of liabilities (V_L) of Uralsib Bank.

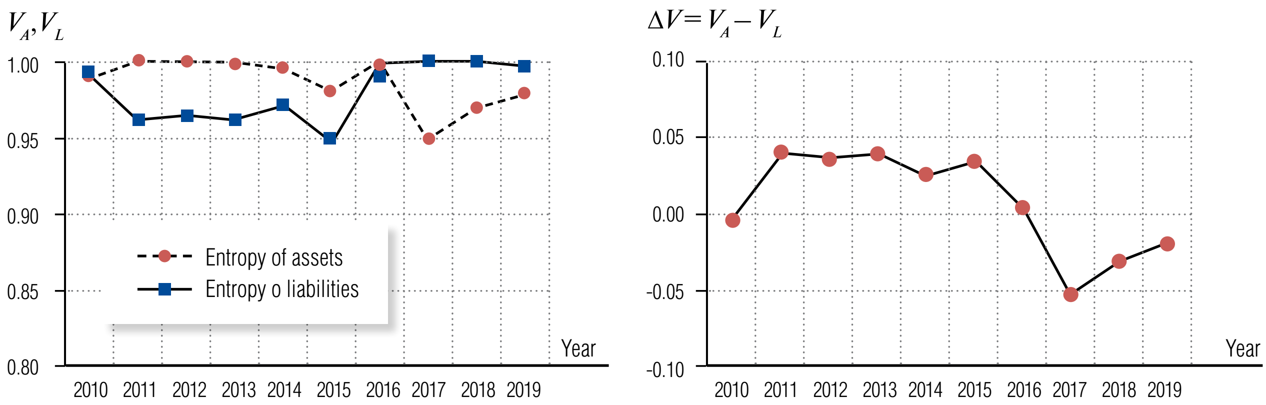

Fig. 10. Entropy of assets (V_A) and entropy of liabilities (V_L) of TEMBR-BANK.

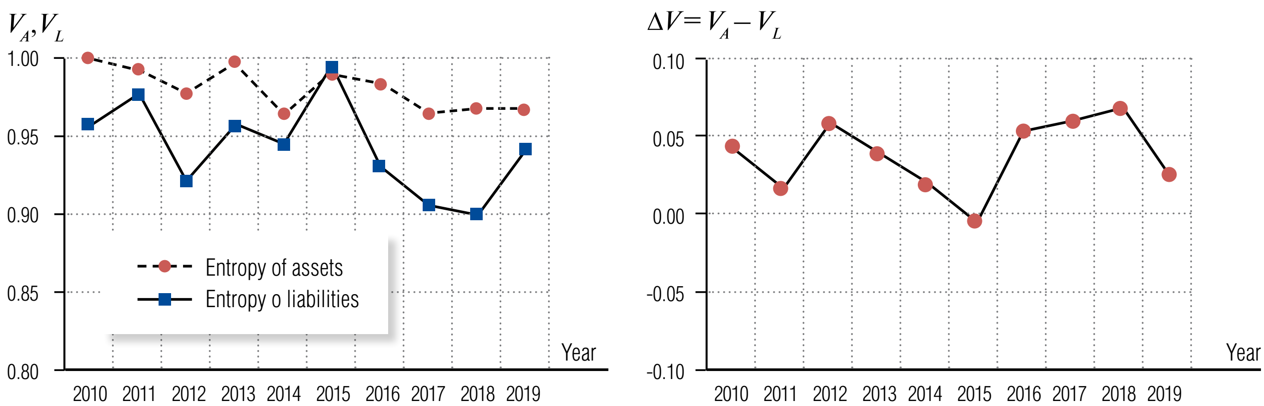

Fig. 11. Entropy of assets (V_A) and entropy of liabilities (V_L) of NVKbank.

Figures 6–11 show characteristic graphs of the time dependencies of the entropy of assets (V_A) and the entropy of liabilities (V_L), calculated by the authors on the basis of data on the balance sheets of [1]:

- licensed banks (*Figs. 6, 7*);
- sanitized banks (*Figs. 8, 9*);
- banks whose license has been revoked (*Figs. 10, 11*).

5. Variances of time dependencies of entropies

An important conclusion of the visual comparison of the time graphs of the entropy of assets and the entropy of liabilities of balance sheets is the obvious presence (*Figs. 6, 7*) or violation (*Figs. 8–11*) of the synchronous behavior of the time dependencies of the entropy of assets (V_A) and the entropy of liabilities (V_L). This fact indicates the expediency of using an estimate of the variance (volatility) of time series of values of the difference in the entropy of assets and liabilities ($\Delta V = V_A - V_L$) as a measure of the spread of the real values of the relative average:

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T}\left(\Delta V_t - \overline{\Delta V}\right)}{T}},$$

where $T = 10$ is the number of observations.

The results of calculating the variances of the time dependencies of the entropy of assets (σ_A), liabilities (σ_L) and the entropy difference ($\sigma_{\Delta V}$) for the period 2010–2019 are shown in *Table 3* for three conditional groups of banks: large, medium and relatively small.

A line-by-line comparison of the variances of the entropy difference ($\sigma_{\Delta V}$) and the status of the banking license gives grounds to assert that existing banks, as a rule, have lower values of the parameter ($\sigma_{\Delta V}$) than those whose license has been revoked.

This statement is not strict in a quantitative sense. That is, the parameter $\sigma_{\Delta V}$ is not a clear criterion that allows you to unambiguously attribute any bank to a particular group. For example, the variance of the entropy difference between assets and liabilities of the rehabilitated bank Uralsib Bank ($\sigma_{\Delta V}$ = 4.55) is higher than that of TEMBR-BANK ($\sigma_{\Delta V}$ = 3.18). However, the license of Uralsib Bank has not been revoked. It is possible that a permit for the right to operate a credit institution is issued taking into account many factors, including, for example, the size of the bank. Nevertheless, the variance of the entropy difference ($\sigma_{\Delta V}$)) can be used as an integral indicator of problems in the activities of specific banks. Probably, at the next stage of research, it would be advisable to study more broadly and in detail the statistics of the limits of the values of $\sigma_{\Delta V}$ for various groups of banks. It can be assumed that a corresponding study of a larger number of banks (for example, all banks that had a license in 2010) will allow us to estimate the values of constants σ and $\overline{\sigma}$ such that if the inequality $\sigma_{\Delta V} < \underline{\sigma}$ is fair, it can be argued that in the medium term the license of the corresponding bank will not be revoked, and if the inequality $\sigma_{\Delta V} > \overline{\sigma}$ is fair, it can be argued that in the medium term, the license of the relevant bank will be revoked.

In addition, in order to more accurately and objectively determine the time, size and source of the imbalance in the bank's activities, it is necessary to separately analyze the time dependencies of the entropies of the item-by-item distributions of assets and liabilities. It is the method of calculating the entropy of line-by-line distributions proposed in this paper that can give an answer to this question. Moreover, it becomes possible to manage the entropy of banks by purposefully adjusting the item-by-item distributions of assets and liabilities.

Conclusion

The application of the entropy approach to the assessment of the equilibrium of the item-by-item distributions of assets and liabilities of the balance sheet significantly expands the boundaries of the practical use of

[1] See: Banking Analyst portal "Analysis of Banks" (https://analizbankov.ru).

Table 3

Variances of time dependencies of entropies and entropy differences of assets and liabilities

Bank	Currency balance on 01.01.2019 (RUB, thou.)	Entropy variance		Variance of the entropy difference $\sigma_{\Delta V}$ (%)	License
		Assets σ_A (%)	Liabilities σ_L (%)		
VTB Bank	14 331 232 043	1.43	1.47	0.47	Have
SberBank	28 361 319 019	3.23	3.11	0.72	Have
Promsvyazbank	1 667 080 707	3.53	2.17	2.88	Rehabilitation
Uralsib Bank	582 185 086	4.64	0.86	4.55	Rehabilitation
TEMBR-BANK	11 636 190	2.21	2.91	3.18	Withdrawn
NVKbank	12 319 149	2.39	6.42	4.34	Withdrawn

Note: In order to correctly perceive the data, we note that the variance of the entropy difference is not an algebraic sum or the average value of the variances of assets and liabilities.

system methods of analysis and management of the state of credit and financial organizations, since a formalized relationship of entropy with the distribution of financial resources by assets and liabilities has been established.

In particular, within the framework of the study, a methodology for calculating entropy as an integral parameter of arbitrary numerical series is proposed and pilot results of its use in analyzing the dynamics of the state of the management system of specific banks are presented. It is shown that a violation of the synchronicity of changes in the entropy of assets and the entropy of liabilities indicates the occurrence of an imbalance in the state of the bank.

Taking into account the unimodal nature of the dependence of entropy on the parameter of uneven distribution, it becomes possible to localize the cause of the imbalance and make targeted adjustments to the distribution of assets and liabilities. ■

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