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Decision support technology for a seller on a marketplace in a competitive environment*

Mikhail G. Matveev 

E-mail: mgmatveev@yandex.ru

Natalya A. Aleynikova 

E-mail: balbashovan@mail.ru

Maria D. Titova 

E-mail: 29_06_titova@mail.ru

Voronezh State University

Address: 1, Universitetskaya pl., Voronezh 394018, Russia

Abstract

This article deals with the problem of improving the effectiveness of a marketplace. The stakeholders of a marketplace are buyers and sellers. The objects are the aggregate of homogeneous products. The effectiveness of the trading platform, which can be characterized by the number of transactions made, will depend on how sufficiently the sellers put up offers. The paper looks at mathematical models to support the decision-making of the seller in making such offers. Focusing not only on the buyer demand but also on the presence of competitors on the site is a distinguishing feature of the models. To describe the competition, the apparatus of game theory is offered, namely the normal form of the game with a bimatrix model with two players: the seller – customer of service and the coalition of other sellers. To match offer and demand, as well as to find the probability of a transaction, fuzzy set theory and aggregation using the Choquet integral are used.

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Keywords: electronic trading platform, marketplace, homogeneous product, linguistic variable, aggregation operator, Choquet integral, bimatrix game, solution in mixed strategies

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Introduction

The development of e-commerce is naturally accompanied by an increase in the level of automation of business processes. At the same time, the emphasis is shifting from automating routine document management processes towards automating more complex decision support processes by e-commerce entities [1–3]. An important aspect of the seller's work in the formation of a product offer is the indicator of the liquidity of the product, which measures the possibility of making a deal with the buyer for a given product offer [4]. Support for the buyer in finding suitable products and sellers is already developing. Examples of the relevant tools can be found in [5]. Automation of this type of business processes for the seller is only in its infancy and there are not enough tools to help the seller [5]. Our work is aimed at increasing the liquidity of the seller's product offer on the marketplace. We propose a numerical measure of the liquidity of a product offer and a new mathematical model for the formation of such an offer (seller's strategy), which in a competitive environment will have high liquidity.

In [6–9], a mathematical model was proposed for formalizing the activities of subjects of an electronic trading platform (ETP) of the marketplace type based on matching demand and acceptable supply [10]. Consumer demand and acceptable seller offers are set parametrically within a single commodity classifier in the form of fuzzy characteristic properties of a homogeneous group of goods. The vector of values of the characteristic properties of a product defines

its differentiation in a group of homogeneous products in terms of a set of characteristics (price, quality, and other characteristic properties) specified both in numerical and categorical formats. The main result of this research is a numerical measure of liquidity, which is formalized as a correspondence between the demand and the allowable supply of the seller for a given set of their characteristic properties (not only for the price, as it takes place in most cases). This makes it possible to compare (on a numerical scale) the correspondences of various product offers to market conditions and select the offer with the best match.

However, [6–9] did not take into account the fact that the seller, putting forward his proposals, is in a competitive environment and the proposals of other sellers can significantly change liquidity. The competition of sellers in the online trading market is appropriately described by game-theoretic models, examples of which are shown in [11–18]. When developing a game-theoretic model, the key tasks are setting the equilibrium conditions, interpreting and calculating the payoff matrix elements, and interpreting the players' strategies. In these papers, game-theoretic modeling of a duopoly is considered with the choice of an equilibrium solution (in the Nash sense) and the interpretation of the outcome of the game in the form of profit. A characteristic feature of these works is the choice of strategies not only in the form of the price of the goods, but also taking into account their quality. Other characteristic properties of the goods are not taken into account.

We believe that the calculated maximum profit is not always achievable in real market conditions. Such a transaction may not take place due to the discrepancy between some properties of the optimal product offer in terms of price and the demand or acceptable offer of the seller. A more realistic goal is a trade-off between maximizing potential profits and being able to conclude a deal. The search for such a compromise consists in maximizing the correspondence between demand and the acceptable product offer along the vector of the characteristic properties of the product, including the price characteristic.

The game situation arises in the form of a seller's competition with a set of other sellers operating in the ETP market and considered as a generalized competitor. The game strategies are set by the acceptable variants of the seller's product offer represented by the corresponding vectors of characteristic properties. The resulting correspondence is interpreted as the subjective probability of making a deal and is proposed as the outcome of the game in game-theoretic modeling of competition in the duopoly market.

The purpose of is paper is to obtain game-theoretic models that make it possible for the seller to form rational offers from an acceptable set of interchangeable types of homogeneous goods.

1. Formalization of the activities of the subjects in ETP

Let the objects of trade on the marketplace be sets of homogeneous goods. A homogeneous product is a set of its interchangeable types, for example, a set of cars of various brands. The types differ in the values of the characteristic properties of a given product, given by the vector of values of the corresponding parameters. Such parameters may be commercial, technical and other possible properties or characteristics of the goods. Let j be the index of the type of a homogeneous product ($j = \overline{1, J}$), its vector of characteristic properties will be denoted as

$$q_j = (q_j^1; \dots; q_j^n; \dots; q_j^N), \tag{1}$$

Here each n coordinate can take values both on a quantitative and on a qualitative scale.

Consumer demand for some kind of homogeneous product can also be formalized in the form of vector g (product characteristics) which is structurally identical to vector q . As a rule, the desires of buyers are vague and approximate. For example, a buyer needs a car with an engine power of 100 to 150 hp. with a cost from the price interval specified by the buyer. Moreover, some values from these intervals are more desirable, some less. Such demand of the buyer can be provided due to the variety of homogeneous goods with different characteristics. It is convenient to represent the coordinates of the consumer demand vector as linguistic variables [19]

$$\tilde{g}_k = (\tilde{g}_k^1; \dots; \tilde{g}_k^n; \dots; \tilde{g}_k^N), \tag{2}$$

where $k = \overline{1, K}$ is the buyer index. The names of linguistic variables coincide with the names of the corresponding characteristic parameters of the description of the type of a homogeneous product. Each variable has piecewise linear membership functions $f_g(x) \in [0; 1]$ whose carriers $x_{\min} \leq x \leq x_{\max}$ reflect the choice of the buyer, and the function values determine the level of his preference [8].

The seller's offer (strategy) represents a specific type of product and is given by a vector similar to (1). It is required to choose the values of the coordinates of q_j in such a way as to ensure high liquidity of the transaction. At the same time, it is necessary to pay attention not only to consumer demand, but also to the permissible possibilities of the seller himself. The values of q_j are limited by the financial and commodity stocks of the seller as well as its functionality. It is assumed that the seller is able to evaluate their functional cost constraints (FCC) and seeks to formulate their offer in such a way as to obtain the maximum compliance with these limitations. FCC are set as

admissible intervals of parameter values with the construction of a membership function over each interval reflecting the preferences of its values. Then the seller's restrictions can be represented by the following vector of linguistic variables

$$\tilde{q} = (\tilde{q}^1; \dots; \tilde{q}^n; \dots; \tilde{q}^N) \quad (3)$$

with the same names of fuzzy characteristics as the demand vector, but with their own membership functions $f_{\tilde{q}^n}(x) \in [0;1]$. Note, that by forming the price membership function in this way, one can set the seller's desire to sell the goods at a high price, i.e., get maximum profit.

Since it is a cumbersome task to track the interaction between the seller and each buyer, we proposed to pass to the generalized consumer demand which can be presented as (2). The membership function for each generalized vector coordinate (2) can be found as a weighted sum:

$$f_{\tilde{g}^n}(x) = \sum_{k=1}^K f_{\tilde{g}_k^n}(x) \cdot w_k \in [0;1]; \quad n = \overline{1, N}, \quad (4)$$

where $w_k = \frac{v_k}{v}$ is the corresponding weight the buyer k , calculated as the ratio of the volume of goods requested by the buyer k to the total volume of commodity demand $v = \sum_{k=1}^K v_k$. Further, we will consider the interaction of the seller with a generalized consumer demand but not with a specific buyer. The validity of this approach is shown in [8].

The range of values of the components of the seller's proposals vector that simultaneously satisfy the generalized demand and the FCC is defined as the intersection of the graphs of the corresponding membership functions for each component pair of the demand vector and the FCC vector [6]. Denote this vector as \tilde{s} . The membership function of the intersection with respect to the component n is determined as

$$f_{\tilde{s}}^n(x) = \min(f_{\tilde{g}}^n(x); f_{\tilde{q}}^n(x)). \quad (5)$$

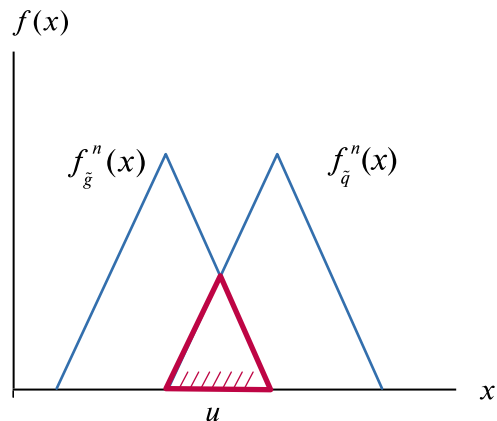


Fig. 1. Graphic illustration of determining the membership function by the component n of the vector \tilde{s} .

Graphical illustration of a possible intersection is shown in Fig. 1.

Carrier $u \subseteq x$ of function $f_{\tilde{s}}^n(x)$ determines the allowable values of the property n of the product; the values of the function determine the degree of compliance of the allowable values of the properties with the generalized demand and the FCC of the seller. The boundaries of the carrier, within which the seller chooses the allowable values of the property n of the product, are calculated as

$$L(f_{\tilde{s}}^n) = \max(L(f_{\tilde{g}}^n); L(f_{\tilde{q}}^n)), \quad (6)$$

$$R(f_{\tilde{s}}^n) = \min(R(f_{\tilde{g}}^n); R(f_{\tilde{q}}^n)), \quad (7)$$

where L and R are left and right boundaries of the carriers, respectively.

Substituting value x^* of the property n of a specific type of the product into function (5), we obtain the degree of local correspondence $f_{\tilde{s}}^n(x^*)$ for the property n . To obtain the conformity of the product to the allowable values for the entire set of properties, it is necessary to aggregate local matches:

$$f_{\tilde{s}} = \text{agr}_n[f_{\tilde{s}}^n(x^*)] \in [0;1], \quad (8)$$

where agr is the aggregation operator for the local matches.

The choice of the aggregation operator is carried out taking into account the specifics of the subject area and the corresponding properties of various operators. An overview of aggregation operators, their properties, and recommendations for use can be found in [20–23]. Particularly, the required property of the aggregation operator is the following expression: $\text{agr}: [0, 1]^n \rightarrow [0, 1]$.

As an aggregation operator, we suggest the discrete Choquet integral with respect to a fuzzy measure [24] which is used when the aggregation result is impacted by the value of each of the properties of the product, as well as if it is necessary to take into account the interaction of properties with each other. An example is the interaction of price and quality of goods. Let us designate the set of product properties as the set of their indices $M = \{n_i\}, i = 1, 2, \dots, N$, and let m be an arbitrary subset of M . The interaction of properties can be taken into account due to the fact that when calculating the Choquet integral, the λ -fuzzy Sugeno measure is used, which is specified on set M and expresses the subjective weight or significance of each subset of properties. It is defined as follows [25]:

$$\varphi(m) = \frac{\prod_{n \in m} (1 + \lambda \varphi_n) - 1}{\lambda} \in [0; 1], m \subseteq M, \quad (9)$$

where φ_n are the coefficients of importance (weights) of individual properties, which can be determined either using special methods or set by an expert [26–29]. The value of λ can be found by solving the following equation

$$\lambda + 1 - \prod_{n=1}^N (1 + \lambda \varphi_n) = 0, \quad (10)$$

with the following condition $\lambda > -1, \lambda \neq 0$.

Then the Choquet integral determining the aggregated correspondence is calculated as follows:

$$\begin{aligned} f_s &= \text{agr}_n [f_s^n(x^*)] = \\ &= \sum_{n=1}^N (f_s^{(n)}(x^*) - f_s^{(n-1)}(x^*)) \times \\ &\times \varphi(m / f_s^i(x^*) \geq f_s^{(n)}(x^*), i \in m), \quad (11) \end{aligned}$$

where $f_s^{(1)}(x^*), f_s^{(2)}(x^*), \dots, f_s^{(N)}(x^*)$ is a permutation of elements $f_s^1(x^*), f_s^2(x^*), \dots, f_s^N(x^*)$ such that

$$f_s^{(1)}(x^*) \leq f_s^{(2)}(x^*) \leq \dots \leq f_s^{(N)}(x^*), f_s^{(m)}(x^*) = 0.$$

We assume, that at $f_s = 0$ the transaction will not take place, but at $f_s = 1$, the transaction will definitely take place. Then the correspondence $f_s \in [0; 1]$ can be interpreted as the subjective probability of making a deal, that is, a numerical measure of liquidity. The concept of subjective probability (hereinafter referred to as probability), based on expert judgment and the use of mathematical methods for processing this judgment are widely used in economic applications, for example in [30, 31].

In the following, the probability of a transaction for an arbitrary seller for the specific product j calculated by formula (8) will be denoted by p_j taking into account that p_j is a function of the strategy q_j of the seller (his specific offer) and is considered as a measure of the liquidity of the product offer.

2. The game model for selecting the seller's offer on the marketplace under competition

So far, we have determined the probability of a trade for a seller provided that there is only one seller on the market. The presence of other competing sellers in the market can significantly change this probability.

Consider the interaction of a seller, a service applicant, with a set of other sellers of some homogeneous product. Let the alternative variants of the seller's product offer (strategy) be represented by the corresponding vectors with different values of the characteristic

properties of a homogeneous product. The choice on a subset of alternative strategies should be made taking into account the competitive offers of the set of other sellers. If the entire set of sellers on the ETP is large enough, then the chosen strategy of one seller that does not dominate in terms of volume will have practically no effect on the choice of sellers from the set. On the contrary, the generalized offer of the set of sellers will significantly change the probability of a transaction of one seller. The seller chooses his strategy for a certain long time period, during which the generalized offer of the set of sellers changes randomly, which leads to a game situation corresponding to the conditions of the bimatrix game.

Two players are considered: a seller (a service applicant) with his own set of strategies $(q_j, j = \overline{1, J})$ and a certain generalized seller, composed of a set of sellers, with his own strategies represented by variants of a generalized offer $(\hat{q}_t, t = \overline{1, T})$.

The payment matrix of the seller is represented by the probabilities of transactions p_{jt} . The payment matrix of the generalized seller is represented by his transaction probabilities \hat{p}_{jt} . Probabilities \hat{p}_{jt} can be defined as follows. A component-by-component generalization of sellers' proposals from the set is performed. Assuming no dominance of individual sellers on the ETP, the generalization for each component of the supply vectors is determined as the average value. The matching of the generalized offer and the generalized demand for each component is performed in the same way as it was done for the individual seller and the generalized demand. The results of the obtained correspondences are aggregated following (8) using the Choquet integral (11). Aggregate matches are considered as probabilities \hat{p}_{jt} of the generalized offer that do not depend on the offers of the seller, i.e., Those elements $\hat{p}_{jt} = \hat{p}_j$.

The formation of generalized strategies is carried out randomly under the assumption of a random nature of the values of the characteristic properties.

The average and standard deviation of each generalized property is determined from a sample of values of the characteristic properties of the offers in a set of sellers. Then a set of strategies, for example, under the assumption that the distributions of random variables are normal, can be obtained using a standard random number generator.

Probability p_{jt} of a transaction for a seller in a competitive environment is obviously a function that depends both on probability of making a transaction p_j with no competition, and on probabilities \hat{p}_{jt} , that is, $p_{jt} = p_{jt}(p_j; \hat{p}_{jt})$. At the same time, we suggest that if probability \hat{p}_{jt} of selling the goods for the generalized seller is less than the similar probability p_j for the seller, then the buyer will buy it from the seller with probability $p_{jt}(p_j; \hat{p}_{jt}) = p_j$. If the probability of selling the product by the generalized seller exceeds p_j for the seller, then the value should decrease, since, most likely, buyers will prefer the goods of the set of sellers with more attractive characteristic values. The probability of sale in this case for the seller can be determined using the following reasoning. Consider a complete group of incompatible events, which includes three situations. The first, when the buyer buys the product from the generalized seller with probability \hat{p}_{jt} ; the second, when the product is bought from the seller (we have to find this probability $p_{jt}(p_j; \hat{p}_{jt})$); and the third, when the product is not bought from either the generalized seller or the regular seller. The probability of the last situation can be defined as $(1 - p_j)(1 - \hat{p}_{jt})$. From the normalization condition

$$p_{jt} + \hat{p}_{jt} + (1 - p_j)(1 - \hat{p}_{jt}) = 1 \tag{13}$$

we have

$$\begin{aligned} p_{jt} &= 1 - \hat{p}_{jt} - (1 - p_j)(1 - \hat{p}_{jt}) = \\ &= p_j - p_j \hat{p}_{jt} = p_j(1 - \hat{p}_{jt}). \end{aligned} \tag{14}$$

Then the probability of a transaction for the seller in a competitive environment is:

$$p_{ji}(p_j; \hat{p}_{ji}) = \begin{cases} p_j - p_j \hat{p}_{ji}, & p_j \leq \hat{p}_{ji}, \\ p_j, & p_j > \hat{p}_{ji}. \end{cases} \quad (15)$$

For example, the seller has a probability of selling 0.21, and the generalized seller has 0.65. Then the probability of selling under competition from the seller is

$$p_{ji}(0.21; 0.65) = 0.21 \cdot (1 - 0.65) = 0.0735.$$

Thus, it is assumed that finite sets of the seller q_j and generalized seller \hat{q}_i strategies are given. The values of the payoff function are given as a bimatrix with elements $A_{ji} = (p_{ji}, \hat{p}_{ji})$. The solution to the problem consists in the rational choice of the strategy (offer) of the seller with a random change in the strategies of the generalized seller.

As a criterion of rationality, we consider the Nash concept of equilibrium [32, 33]. A set of mixed strategies $q^* = (q_j^*; \hat{q}_i^*)$ is called as the Nash equilibrium situation in mixed strategies if the choice of any side of the mixed strategy other than one which leads to one of the inequalities

$$V_1(p_{ji}; \hat{p}_{ji}^*) \leq V_1(p_{ji}^*; \hat{p}_{ji}^*) \quad (16)$$

or

$$V_2(p_{ji}^*; \hat{p}_{ji}^*) \leq V_2(p_{ji}; \hat{p}_{ji}^*). \quad (17)$$

where V_1, V_2 are the mathematical expectations of the winnings of the seller and the generalized seller, respectively.

The above inequalities indicate that a deviation from the equilibrium situation by one side cannot increase its payoff.

3. Numerical example

Using the example of one seller and three buyers, we calculate the equilibrium mixed strategy of the seller. Let a homogeneous product be characterized by three

parameters (properties). The values of the parameters characterizing the FCC of the seller are given in the form of triangular fuzzy numbers in *Table 1* indicating the left boundary of the carrier, the mode and the right boundary of the carrier, respectively. The carrier is normalized in the range from 0 to 1.

Table 1.

FCC of the seller given as triangular fuzzy numbers

1-st parameter	2-nd parameter	3-rd parameter
(0.4; 1; 1)	(0.2; 0.4; 1)	(0.5; 1; 1)

For buyers, the initial data is given in *Table 2*.

To determine the values of the membership function of the generalized consumer demand for each parameter separately, we use (4). To do this, we divide interval [0, 1] into 10 parts and at each point determine the value of the membership function of the generalized consumer demand $f_{\hat{g}^n}(x), n=1,3$.

For example, at $x = 0.3$ the value of the membership function by the first parameter for the first buyer is 0.5, for the second 0.25, for the third 0.833. Weights are $w_1 = 10 : (10 + 4 + 6) = 0.5, w_2 = 0.2, w_3 = 0.3$. Then, using (4) we have

$$f_{\hat{g}^1}(0.3) = 0.5 \cdot 0.5 + 0.25 \cdot 0.2 + 0.833 \cdot 0.3 = 0.55.$$

In *Fig. 2* we show the membership function for the generalized demand by each parameter.

Next, we determine the boundaries and membership functions of the components of the fuzzy vector of the seller's proposals that simultaneously satisfy the generalized demand and FCC following to (5)–(7). In this case, membership functions $f_{\hat{g}^n}(x), n=1,3$ by each parameter are shown in *Fig. 3*.

Let the seller offer three goods (strategies) with parameters written as vectors, the coordinates of which are normalized $q_1 = (0.5; 0.4; 0.8), q_2 = (0.4; 0.6; 0.9),$

Table 2.

Characteristics of consumer demand given by triangular fuzzy numbers

Buyer	The volume of goods requested by the buyer	1-st parameter	2-nd parameter	3-rd parameter
1	10	(0.2; 0.4; 1)	(0.2; 0.5; 1)	(0; 0.4; 0.6)
2	4	(0.2; 0.6; 1)	(0.5; 1; 1)	(0.4; 0.4; 1)
3	6	(0.2; 0.2; 0.8)	(0.2; 0.6; 1)	(0; 0; 1)

$q_3 = (0.8; 0.5; 0.5)$. Then, substituting the corresponding coordinates in functions $f_s^n(x), n = \overline{1,3}$, we obtain the local correspondences $f_s^n(x^*), n = \overline{1,3}$. For the first strategy, we get the vector of local correspondences (0.167; 0.483; 0.1267); for the second one we have (0; 0.267; 0.725); for the third one we have (0.267; 0.725; 0).

Now, we aggregate the local correspondences using the Choquet integral (9)–(11). To determine λ from equation (10), we set the coefficients of importance of the product parameters equal to $\varphi_1 = 0.3, \varphi_2 = 0.6, \varphi_3 = 0.2$. Then, equation (10) takes the form

$$\lambda + 1 - (1 + \lambda \cdot 0.3) \cdot (1 + \lambda \cdot 0.6) \cdot (1 + \lambda \cdot 0.2) = 0,$$

at $\lambda > -1, \lambda \neq 0$. Root of the equation is $\lambda = -0.286$.

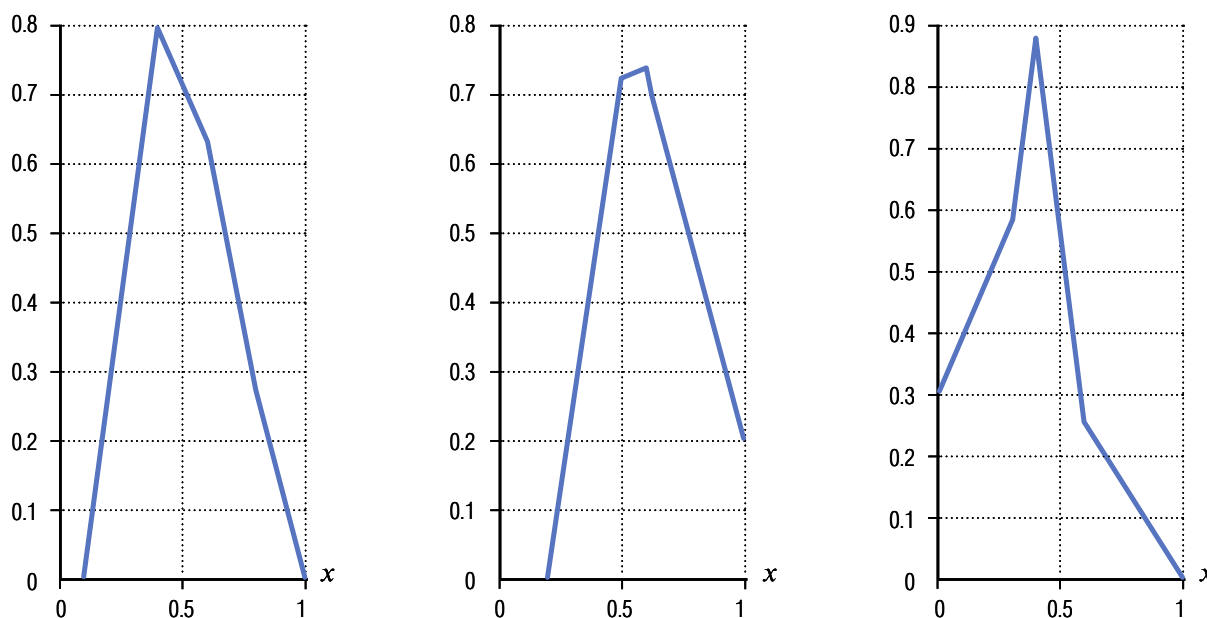


Fig. 2. The membership function of the generalized demand by each parameter.

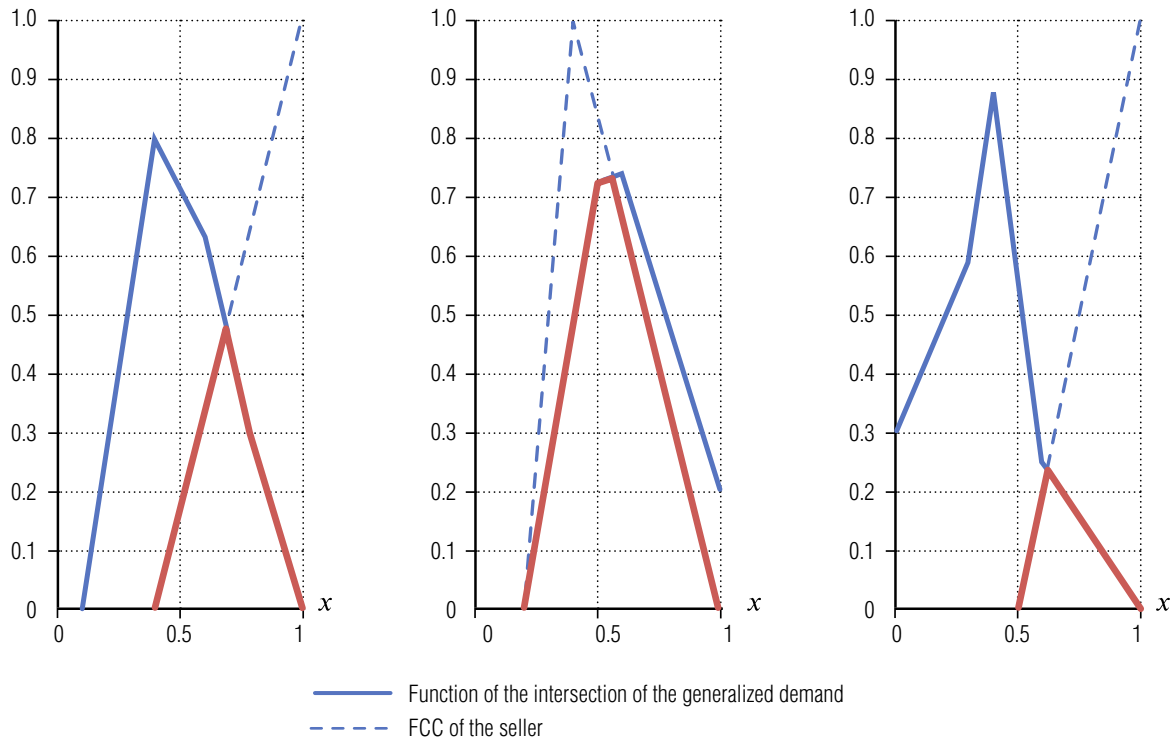


Fig. 3. Membership functions of the intersection of the generalized demand and the FCC of the seller by each parameter.

Determine the Choquet integral for the first strategy of the seller. Arrange the coordinates of the vector of local correspondences in ascending order $f_s^{(1)} = f_s^3 = 0.1267$, $f_s^{(2)} = f_s^1 = 0.167$, $f_s^{(3)} = f_s^2 = 0.438$. Then,

$$f_s = (f_s^{(1)} - f_s^{(0)}) \cdot \varphi(\{1, 2, 3\}) + (f_s^{(2)} - f_s^{(1)}) \cdot \varphi(\{1, 2\}) + (f_s^{(3)} - f_s^{(2)}) \cdot \varphi(\{2\}),$$

where

$$\varphi(\{1, 2, 3\}) = \frac{(1 - 0.286 \cdot 0.3) \cdot (1 - 0.286 \cdot 0.6) \cdot (1 - 0.286 \cdot 0.2) - 1}{-0.286} = 1$$

(equation (9)),

$$\varphi(\{1, 2\}) = \frac{(1 - 0.286 \cdot 0.3) \cdot (1 - 0.286 \cdot 0.6) - 1}{-0.286} = 0.8485,$$

and $\varphi(\{2\}) = 0.6$.

As a result,

$$\begin{aligned} f_s &= (f_s^{(1)} - f_s^{(0)}) \cdot \varphi(\{1, 2, 3\}) + \\ &+ (f_s^{(2)} - f_s^{(1)}) \cdot \varphi(\{1, 2\}) + (f_s^{(3)} - f_s^{(2)}) \cdot \varphi(\{2\}) = \\ &= (0.1267 - 0) \cdot 1 + (0.167 - 0.1267) \cdot 0.8485 + \\ &+ (0.438 - 0.167) \cdot 0.6 = 0.35. \end{aligned}$$

Similarly, for the second strategy, the Choquet integral is $f_s = 0.41$, for the third strategy is $f_s = 0.501$.

We have the probabilities of the transaction for the seller by each product:

$$p_1 = 0.35, p_2 = 0.41, p_3 = 0.501. \tag{18}$$

Now consider a generalized seller which is competitor for the seller offering a homogeneous product.

Suppose that a generalized seller has three strategies $\hat{q}_t, t=1,3$, and the probabilities of a deal for each of them are $\hat{p}_1 = 0.414, \hat{p}_2 = 0.374, \hat{p}_3 = 0.264$.

Probabilities (18) for the seller has been obtained in the case when there is no competition. Under competition for seller, it is necessary to obtain the values of (15) that are substituted into the bimatrix as his payoffs.

Because $p_1 = 0.35 < \hat{p}_1 = 0.414$, then, using (15) $p_{11} = 0.35 - 0.35 \cdot 0.414 = 0.205$. At the same time $p_{13} = 0.35$ because $p_1 = 0.35 > \hat{p}_3 = 0.264$. Other winnings of the seller are calculated similarly.

As a result, the bimatrix game takes the form:

$$\begin{pmatrix} & \hat{q}_1 & \hat{q}_2 & \hat{q}_3 \\ q_1 & (0.205; 0.414) & (0.219; 0.374) & (0.35; 0.264) \\ q_2 & (0.240; 0.414) & (0.41; 0.374) & (0.41; 0.264) \\ q_3 & (0.501; 0.414) & (0.501; 0.374) & (0.501; 0.264) \end{pmatrix}. \quad (19)$$

Note, that we are only interested in the choice of the seller strategy.

The solution to the bimatrix game using the Nash equilibrium strategy in the situation of mixed strategies [32, 33] gives the equilibrium in pure strategies for the seller with the price of the game 0.501. That is, the seller should put up for sale the first product with characteristics (0.5; 0.4; 0.8); the probability of sale under competition, will be equal to 0.501. For the generalized seller, equilibrium is reached in pure strategies with the price of the game 0.414.

Note that the method for calculating the Nash equilibrium is quite cumbersome and its computational complexity increases with the increase in the dimension of the problems being solved.

The above result can be obtained using a simpler technique. It is shown in [34] that in the game 2×2 the same result can be obtained by each side only based on their own payoff matrices. To do this, it is neces-

sary to split the bimatrix game into two ordinary zero-sum matrix games. Each player can calculate from the matrix of his own payoffs the optimal average payoff, which coincides with the payoff in the equilibrium situation; using his own matrix, the player can find the optimal strategy of the other player, but not his own. In our case, consider the matrices 3×3 :

$$A = \begin{pmatrix} 0.205 & 0.219 & 0.35 \\ 0.240 & 0.41 & 0.41 \\ 0.501 & 0.501 & 0.501 \end{pmatrix}$$

$$\text{and } B = \begin{pmatrix} 0.414 & 0.414 & 0.414 \\ 0.374 & 0.374 & 0.374 \\ 0.264 & 0.264 & 0.264 \end{pmatrix}. \quad (20)$$

Find the solution to the matrix game in mixed strategies for the generalized seller using matrix A . To do this, we denote by $(\alpha_1, \alpha_2, \alpha_3)$ the vector of probabilities of applying the corresponding strategies by the generalized seller, and by v – the price of the game. Substituting

$$x_1 = \frac{\alpha_1}{v}, x_2 = \frac{\alpha_2}{v}, x_3 = \frac{\alpha_3}{v}.$$

we compose a linear programming problem:

$$F = x_1 + x_2 + x_3 \rightarrow \max,$$

under restrictions

$$\begin{cases} 0.205x_1 + 0.219x_2 + 0.35x_3 \leq 1, \\ 0.240x_1 + 0.41x_2 + 0.41x_3 \leq 1, \\ 0.501x_1 + 0.501x_2 + 0.501x_3 \leq 1. \end{cases}$$

Solutions to this problem are $x_1 = 1.996, x_2 = 0$ and $x_3 = 0$. The game price is

$$v = \frac{1}{x_1 + x_2 + x_3} = \frac{1}{1.996} = 0.501.$$

When passing to probabilities, we get $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0$. Therefore, the solution in pure strategies for the generalized seller is (1; 0; 0). The game price for the

seller is 0.501. The pure strategies obtained for the generalized seller and the game price for the seller coincide with the strategies and the game price that were found when solving the bimatrix model using the Nash equilibrium technique.

Obtain the solution to the matrix game in mixed strategies for the seller, using matrix B . To do this, we denote by $(\beta_1, \beta_2, \beta_3)$ the vector of probabilities of applying the corresponding strategies by the seller, and by v – the price of the game. Then, the linear programming problem for solving the game in mixed strategies, taking into account the change of variables, as in the previous model, has the form:

$$F = x_1 + x_2 + x_3 \rightarrow \max,$$

under restrictions

$$\begin{cases} 0.414x_1 + 0.414x_2 + 0.414x_3 \leq 1, \\ 0.374x_1 + 0.374x_2 + 0.374x_3 \leq 1, \\ 0.264x_1 + 0.264x_2 + 0.264x_3 \leq 1. \end{cases}$$

Solutions to this problem are $x_1 = 2.4, x_2 = 0, x_3 = 0$. The game price is

$$v = \frac{1}{x_1 + x_2 + x_3} = \frac{1}{2.4} = 0.414.$$

Therefore, the solution in pure strategies for the seller is $(1; 0; 0)$. That is, in a competitive environment, the optimal strategy for him is the first one. The

price of the game for the generalized seller is 0.414. The resulting solution also coincides with the solution which has been obtained for a bimatrix game using the Nash equilibrium technique.

Thus, we can simplify the procedure for finding a Nash equilibrium in mixed strategies by reducing the solution of a bimatrix game to solving two zero-sum games with payoff matrices (20).

Conclusion

The research we conducted made it possible to obtain a set of decision support models for the seller in the formation of a product offer for a homogeneous product on the marketplace in a competitive environment. The formation of the proposal is carried out in two stages. First, the seller, receiving information about the generalized demand, and knowing his functional and cost limitations, using the proposed models, can determine the permissible ranges of values for the characteristics of a homogeneous product which provide non-zero liquidity. Based on them, he can form alternative versions of his product offerings (product strategies). The choice of a product strategy in a competitive environment is carried out within the game-theoretic duopoly model using the Nash criterion. The example shows that it is possible to simplify the procedure for finding a Nash equilibrium in mixed strategies by reducing the solution of a bimatrix game to the solution of two zero-sum games. ■

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About the authors

Mikhail G. Matveev

Dr. Sci. (Tech.), Professor;

Head of Department of Information Technologies in Management, Voronezh State University, 1, Universitetskaya pl., Voronezh 394018, Russia;

E-mail: mgmatveev@yandex.ru

ORCID: 0000-0002-6528-6420

Natalya A. Aleynikova

Cand. Sci. (Physics and Mathematics);

Associate Professor, Information Technologies in Management, Voronezh State University, 1, Universitetskaya pl., Voronezh 394018, Russia;

E-mail: balbashovan@mail.ru

ORCID: 0000-0001-5967-8260

Maria D. Titova

Master student, Information Technologies in Management, Voronezh State University, 1, Universitetskaya pl., Voronezh 394018, Russia;

E-mail: 29_06_titova@mail.ru

ORCID: 0000-0002-4647-7803