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Mathematical model of the formation of supply chains of raw materials from a commodity exchange under conditions of uncertainty*

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Abstract

The formation of raw material supply chains is very closely related to production problems at a timber processing plant. Since the beginning of the second industrial revolution, one urgent question has been the formation of supply chains for raw materials and the optimal calculation of production volumes for each individual day. This article examines a forestry enterprise that does not have its own sources of wood, which daily solves the problem of ensuring the supply of raw materials and optimal production load. A commodity exchange is considered as a source of raw materials where lots randomly appear every day in different raw material regions. In the scientific literature, there are many approaches to calculating

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the optimal profit value over the entire planning horizon, but they do not consider many features that are important for a timber processing enterprise. This paper presents a mathematical model which is a mechanism for making daily decisions over the entire planning horizon and differs in that it allows one to take into account the share of useful volume and the delivery time of raw materials under conditions of uncertainty. The result of the model is the optimal profit trajectory, considering the volume of raw materials, the delivery time of lots, the volume of profit and the production volume of goods. The model was tested on data from the Russian Commodity and Raw Materials Exchange and one of the Primorsky Territory enterprises. Analysis of the results showed that there are difficulties in planning supply chains and production volumes. An assessment of the optimality of raw material regions was carried out. The advantages and disadvantages of the mathematical model are formulated.

Keywords: production optimization, transport problem, forest industry, commodity exchange, supply chains, product release

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Introduction

The wood supply chain plays a vital role in the global economy, providing essential raw materials to various industries such as construction [1–4], furniture [5–7] and paper [8]. Supply chains are complex and dynamic; their formation depends on the efficiency of forestry management, processes of harvesting, wood processing, distribution and consumption of wood products. Effective supply chain management (SC, SCM) of wood is critical to ensure sustainability of production and conservation of resources while promoting economic growth.

In recent years, there has been increased interest in wood supply chain modeling to optimize and improve production sustainability [9–11]. Modern SC formation models are developed to provide management with valuable information about supply chain performance and can be used to improve the sustainability and efficiency of management strategies [12, 13]. There is a significant amount of research, the scope of which lies in various areas of forestry production: forest management and the processes of logging, processing, distribution and consumption.

Modeling wood supply chain management processes is critical to efficiently use resources, reduce environmental impact and improve economic performance. As technology advances, data analysis techniques such as machine learning and stochastic optimization are being used to develop models to provide more accurate predictions and effective decision-making tools.

1. Literature review

The wood supply chain is a complex network of processes including forest management, harvesting, transportation, processing and distribution. Effective wood supply chain management is critical to optimizing operations and achieving the sustainability goals of any forest products business. Mathematical modeling is a powerful tool to support decision-making processes and optimize various aspects of wood supply chain management. Let's consider the experience of modeling the problem of supply chain management.

1.1. Timber harvesting optimization models

Optimizing timber harvesting processes is essential for sustainable forest management and maximizing economic returns. Mathematical models such as Mixed-Integer Linear Programming (MILP) models are widely used to determine the optimal timing and spatial distribution of logging. Such models consider various factors, including wood growth, market demand, operating costs and environmental constraints, to support decision-making processes. For example, in [9], the authors propose a MILP model that optimizes timber harvesting and road construction operations in several logging areas, considering economic, environmental and transportation factors. The model aims to maximize the net present value of timber revenues while minimizing logging costs and environmental impacts.

Efficient logistics and transportation are critical components of the wood supply chain, ensuring timely delivery of wood products and minimizing transportation costs. Mathematical modeling methods are used to optimize transport routes, vehicle planning and inventory management. The authors of the study [10] developed an MILP model for multi-day truck routing. The model was applied to a Brazilian logging company that operated one factory and several logging sites equipped with cranes. The model considers the Less Than Truckload (LTL) problem, allowing for repeated movement of trucks between several forest apiaries. The objective function aims to minimize transportation costs, the number of trucks, the number of trips and overtime costs. The model was applied to a case with 5 available harvesters and 48 trucks. Methods that directly use optimization models are sensitive to the dimension of the problem in terms of the number of variables and constraints. In addition, such models usually require assumptions, for example about full trucks, as in [10], to be able to formulate a mathematical model of the problem being solved.

1.2. Inventory and supply chain management models

There are many models in the literature dedicated to the development and application of various approaches to solving a variety of problems in the field of inventory and supply chain management. For example, intuitionistic fuzzy sets (hereinafter IFS) were used [14, 15]. Measures of possibility, necessity and reliability are used as a new approach to solving intuitionistic fuzzy optimization problems [16, 17]. They are also applied in manufacturing, stock-out inventory models to obtain Pareto optimal solutions [18]. Multi-objective IFS optimization has been used, for example, in studies [19–21].

In addition, dynamic programming methods were also used to optimize multi-echelon supply chain problems. Thus, a neurodynamic programming model was developed in [22] to solve the two-stage problem of inventory optimization under conditions of demand uncertainty. Testing of the model in practice showed a reduction in enterprise costs by 10%. The authors of the study [23] formulated the supplier-managed inventory routing problem as a Markov decision process and applied the approximate dynamic programming method to solve it. The authors of [24] developed an approximate dynamic programming (ADP) model based on Lagrangian relaxation for inventory management of a network with one product and several sites. The authors of [25] use ADP methods and apply stochastic approximation to calculate optimal underlying inventory levels given news provider problems over the horizon of multiple periods of backlogs and lost sales. The authors of the study [26] apply ADP methods to solve the problem of inventory management at several enterprises and with a given number of products, considering the variability of some processes.

Reinforcement learning has also been applied to the Inventory Management Problem (IMP) [27]. Thus, the team of authors in [28] use Q-learning for a four-stage IMP with a 12-week cycle and non-stationary demand. In [29], the authors trained the Deep Q-Network neural network architecture to achieve near-optimal results in the Beer Game, a classic example of a multi-layer IMP. The authors of [30] uses Q-learning methods and the SARSA model for optimal replenishment of perishable goods.

1.3. Conclusions and formulation of the research problem

Modeling has become an important tool for analyzing and improving supply chains in various manufacturing areas. The complexity of the supply chain, along with increasing pressure to minimize environmental impact, implement sustainable practices and consider social and operational considerations, has led to the development of a wide range of models. In the wood supply chain, modeling can optimize the flow of raw materials from their origin to their destination by minimizing costs, reducing environmental impact and increasing efficiency.

As a review of the literature showed, there are many works devoted to the subject of SCM, however, many models and approaches are not applicable in practice when managing a forestry enterprise in matters of forming supply chains in conjunction with determining production volumes.

It is worth noting that the existing models are affected by the failure, firstly, to take into account the coefficient of the useful volume of raw materials that will reach the warehouse. This must first be separated from rot, and then processed into dust. Then one uses a press to produce OSB boards. Secondly, there are the tools of daily decision-making based on the supply of lots of raw materials on the exchange that day. For example, work [12] is devoted to a similar problem, but does not take into account the considered feature with the useful volume of raw materials that will reach the warehouse. In addition, there is the absence of a qualitative forecast of the situation on the commodity exchange (when and in what volume lots will appear for sale on the stock exchange) and it is not at all applicable in practice. Work [11] is devoted to the same problem, but with an emphasis on pricing of final goods. Here, as in [12], there is no consideration of the coefficient of the useful volume of raw materials that can reach the warehouse. In addition, the second negative side of the model is repeated: there is no possibility to make a daily decision based on the current offer on the stock exchange. However, it is worth noting that the problem of considering the coefficient of useful volume of raw materials is not new and was considered in [13]. This article is devoted to assessing the optimal value of profit over the entire planning horizon of the forestry production cycle, namely: SCM and calculation of production volumes, where the flow of raw materials is carried out from the commodity exchange. However, there is still no clear description of how to use the model so that the enterprise can make decisions related both to the problems of purchasing raw materials from the exchange and to calculating production volumes so that the profit value at the end of the planning horizon is maximum and at the same time extremely close to optimal.

Thus, the problem of managing an enterprise in the timber industry remains unresolved, when the enterprise does not have its own sources of raw materials for production, and when every day a decision has to be taken on the formation of the flow of raw materials from the commodity exchange and at the same time on the volume of finished products over the entire planning horizon, maximizing the value of the total profits, in conditions of uncertainty in the supply of lots on the stock exchange and taking into account the coefficient of useful volume of raw materials, technology of production of goods and the specifics of logistics of raw materials to the warehouse. This work is devoted to the development of a model that solves the current problem.

2. Research purpose, objectives and hypothesis

Considering the processes of enterprise functioning, the most important for forestry production are the formation of supply chains for raw materials and the volume of production of goods.

Considering the sources of raw materials entering the stock exchange, the exchange enters into agreements with tenants of forest plots from various regions on the use of the trading platform. After completing an exchange transaction between the raw material processing enterprise (timber industry complex) – the customer and the tenant of the forest plots (logger) – the seller, the volume of raw materials stated in the contract is sent to the customer. It is only possible to buy a lot on the exchange in its entirety [11–13, 31–33].

The purpose of this study is to develop a mathematical model that allows the formation of supply chains of raw materials from the commodity exchange in conditions of supply uncertainty and differs from already known models in that, firstly, the decision-making process is carried out daily, and, secondly, it allows you to take into account the likely time of delivery[1] of raw materials to the warehouse and changes in the working volume of raw materials which depends on external factors (temperature, insects, etc.).

Research objectives:

1. Construction of an economic and mathematical model.
2. Designing an algorithm for finding a solution for the developed model.
3. Analysis of model testing results.

It is well known that such a problem can be solved optimally when all the values that were played (lots, travel time) are already known. However, there is no understanding whether it is possible to make decisions every day on the formation of supply chains of raw materials and on production volumes so that the profit value is as close to the optimal value as possible. We assume that a model which allows you to solve the problem as close as possible to the optimal one exists, where decisions are made every day, having only the data of the current day and many assumptions about what the situation will be "tomorrow."

3. Mathematical model[2]

Any production, including the timber industry, is unable to function without the existence of a source of raw materials. To ensure the supply of the required volume of raw materials, it is necessary to identify wood suppliers. To do this, we will use the services of the St. Petersburg International Commodity and Raw Materials Exchange (SPbIMRME)[3]. Every day the stock exchange publishes data on how many transactions (orders) were made, at what price and what volume of raw materials was sold. In addition, the exchange provides services for the delivery of raw materials to the consumer, which is also included in the price of the goods. The exchange represents many regions from which raw materials could potentially come [11–13, 33]. We will specifically change or expand the input data for solving the problem to make it more difficult for the model to find a solution.

After a sufficient volume of raw materials has arrived at the production warehouse, the enterprise must decide on the optimal vector to produce the final product, focusing on the maximum possible production volume [11–13, 33].

Let's consider the scheme for purchasing raw materials and calculating production volumes. It is known that profit maximization at an enterprise is achieved if and only if the necessary quantities are calculated in a single model. That is to say, the model considers both the volume of production and the flow of raw materials. Since an enterprise usually does not know what will happen on the market (exchange) tomorrow, it plans only today based on the estimated situation at the enterprise "tomorrow."

This raises the question of which planning period $\tilde{T} \geqslant 1$ to choose. In this work we will set it to one value to solve the entire problem.

We introduce some assumptions. Let, firstly, the enterprise know for a certain value of E periods what lots were drawn on the stock exchange and the situation with workload on the railways (railroads). Secondly, we also assume that the situation on the raw

[1] The purchase and sale agreement specifies the methods and price of timber delivery. Delivery can be carried out by the enterprise, but further we will consider the delivery of raw materials by the supplier.

[2] The software implementation of the developed model and exchange sales statistics can be found at the link https://drive.google.com/drive/folders/1THzU7BHjGgpUgZiXQbvJNaIA14biWO1t?usp=sharing

[3] https://spimex.com/

materials market does not change much over the years. Then the enterprise, based on these data, with information at one's enterprise for the same periods, can build a model to find the optimal solution. Then there is an opportunity to get many optimal trajectories of profit values and volumes of raw materials in the warehouse every day over the entire planning horizon. In this work, we focus on considering the trajectory of raw material inventories in a warehouse.

In this case, it is possible to build a regression that would reflect the average expected value in aggregate for all types of raw materials in warehouse $\tilde{b}_m$ on each individual day m over the entire planning horizon of M days, depending on which lots are available today.

Since the solution has to be sought on a certain interval $[m, \min(M, m+\tilde{T})]$, and the value of the expected total volume of raw materials is known only for the current day m $\tilde{b}_m$, the question arises about what volume of raw materials in the warehouse we expect in the next days $[m+1, \min(M, m+\tilde{T})]$, where $m+1 \neq M$.

Returning to already found optimal trajectories of the total volume of raw materials in the warehouse in the previous E periods, from here you can calculate the value

$$\tau(m)=\begin{cases}\dfrac{\sum_{e=1}^{E}\dfrac{\sum_l b_{lm}(e)}{\sum_l b_{l(m-1)}(e)}}{E}, m<M,\\ 0,\ m=M\end{cases}$$

where $\sum_l b_{lm}(e)$ — this is the total volume of raw materials for all their types l on day m for each individual input data sample $e \in E$. Then we will search for a solution to the current problem that would take into account the forecast value $\tilde{b}_m$ on the current day m and some correction for the entire planning period forward $\min(M-m, \tilde{T})$

$$\prod_{\underline{t}=1}^{t}\tau(m+\underline{t}),\ t=0:\min(M-m, m+\tilde{T}).$$

Let's introduce the following set of parameters and variables.

Options:

p_{km} — price for product type k on day m;

c_{ilrm} — price of lot i with type of raw material l from region r, appearing on the exchange on day m;

A_{lk} — rate of consumption of raw materials of type l for the production of a unit of goods of type k;

$\gamma_{\tilde{m}m}$ — spoilage rate of raw materials purchased on day $\tilde{m}$ to day m ($m \geqslant \tilde{m}$);

V_{ilrm} — volume of raw materials in lot i with raw material type l from region r, appearing on the exchange on day m;

H_{nk} — maximum production volume of goods of type k on day m;

$\underline{b}$ — emergency level of raw material reserves;

$\overline{b}$ — maximum warehouse capacity;

B_0 — initial budget level;

FC — fixed costs;

M — planning horizon;

L_r — distance from warehouse to region r;

S_m — distance traveled by the application on day m;

π_m — profit value at the moment day m;

$\varepsilon^{(3)}$ — noise (random variable) component of the working volume of raw materials that reached the warehouse;

left and *right* — the minimum and maximum value of a random variable distributed according to a uniform law;

$LN(a_m, \delta_m)$ — log normal distribution of a random variable with parameters (a_m, δ_m) respectively;

$\tilde{T}$ — the period for which the enterprise solves the task $F_m^{(1,2)}$ (days);

E — number of different sets of input parameters $\{V_{ilrm}(e), c_{ilrm}(e), T_{r\tilde{m}}(e)\}$.

Let's consider the notation with and without a tilde above the parameter. We will assume that the value with a tilde above the variable is the value that the enterprise evaluates, and without the tilde is its real value. For example,

$T_{r\tilde{m}}$ – time during which a lot purchased on day $\tilde{m}$ from region r will reach the warehouse;

$\tilde{T}_{r\tilde{m}}$ – time during which, according to the enterprise's estimates, a lot purchased on day $\tilde{m}$ lot from region r will reach the warehouse.

Variables:

x_{km} – volume of production of goods of type k on day m;

λ_{ilrm} – fact of purchase of lot i with type of raw material l from region r, which appeared on the exchange on day m;

b_{lm} – level of stock of raw materials of type l in the warehouse on day m;

$b_{lm}(e)$ – the value of the stock of raw materials of type l on day m, which was found when solving the problem on data e.

Task $F_m^{(1,2)}$ takes the form (1–18). Objective function (1) is aimed at maximizing profit values on each day $m + t - 1$:

$$\sum_{t=1}^{\tilde{T}}\left(\sum_k p_{k(m+t-1)}x_{k(m+t-1)} - \sum_{j=1}^{2}\sum_l N^{(j)}\varepsilon_{l(m+t-1)}^{(j)}\right) - \\ -\sum_{i,l,r} c_{imrl}\lambda_{imrl} \to \max. \quad (1)$$

Relation (2) specifies the relationship between the volumes of raw materials in the warehouse, the volume of raw materials spent in production and the volumes of raw materials that reached the warehouse:

$$b_{l(m+t-1)} - b_{l(m+t-2)} + \sum_k A_{lk}x_{k(m+t-1)} - \\ -\tilde{\gamma}_{\tilde{m}(m+t-1)}\sum_{i,r} V_{i\tilde{m}rl}\lambda_{i\tilde{m}rl} + \sum_{j=1}^{2}(-1)^j\varepsilon_{l(m+t-1)}^{(j)} = 0, \quad (2)$$

where

$t = 1{:}\,\tilde{T}$;

$\tilde{T} = \text{const}$;

$\tilde{T} \geq \max(\tilde{T}_{r\tilde{m}})$;

$N^{(j)} >> 1$;

$j = 1{:}\,2$;

$b_{l(m-1)} = \text{const}$;

$\tilde{T} = \min(\tilde{T}, M - m + 1)$;

$\tilde{m} + \tilde{T}_{r\tilde{m}} = m + t - 1$.

Constraints (3–4) set possible limits for the values of variables responsible for the volume of production of each type of product on each day (3) and when the fact of purchasing raw materials on the exchange is a variable, and in which cases it is considered a constant (4). The $\lambda_{i\tilde{m}rl}$ variable responsible for the fact of making a decision to purchase lot i on day $\tilde{m}$ from region r with raw material type l is a constant if and only if $\tilde{m} \leq m$, where $\tilde{m}$ is the date of appearance of the lot in question on the exchange, and m is the day of acceptance solutions, otherwise is a variable:

$$x_{km} \in N, \quad (3)$$

$$\lambda_{i\tilde{m}rl} = \begin{cases} \text{const}, \ \tilde{m} \leq m \\ \{0;1\}, \begin{cases} \tilde{m} = m \\ \tilde{T}_{r\tilde{m}} = t - 1. \end{cases} \end{cases} \quad (4)$$

Constraint (5) is aimed at ensuring that the search for a solution is based on the required total volume of raw materials in the warehouse. At the end of the planning horizon, we will assume that the enterprise stops its production, so it does not require raw materials in the warehouse:

$$\sum_l b_{l(m+t-1)} = \begin{cases} \min\left(\bar{b}, \prod_{\underline{t}=1}^{t}\tau(m+\underline{t})\cdot\tilde{b}_m\left(\{V_{imrl}\}_{i,r}, \{c_{imrl}\}_{i,r}, m\right)\right), \\ t \in \left(1, \tilde{T}\right) \\ 0, t = M - m + 1, \end{cases} \quad (5)$$

where

$$\sum_l b_{lm} = \tilde{b}_m\left(\{V_{imrl}\}_{i,r}, \{c_{imrl}\}_{i,r}, m\right);$$

$$\tau(m) = \begin{cases} \dfrac{\sum_{e=1}^{E}\dfrac{\sum_l b_{lm}(e)}{\sum_l b_{l(m-1)}(e)}}{E}, m < M \\ 0, m = M. \end{cases}$$

Inequality (6) states that there cannot be a situation where the volume of raw materials in the warehouse drops below zero, taking into account of penalty variables $\varepsilon_{l(m+t-1)}^{(j)}$:

$$b_l(m+t-1)+\sum\nolimits_{j=1}^{2}(-1)^j \varepsilon_{l(m+t-1)}^{(j)} \geq 0. \quad (6)$$

Inequality (7) reflects the current level of the enterprise budget on each individual day $m+\underline{t}-1$ during whole planning horizon $m+t-1$:

$$\pi_{m-1}+\sum\nolimits_{\underline{t}=1}^{t}\left(\sum\nolimits_k p_{k(m+\underline{t}-1)} x_{k(m+\underline{t}-1)}\right)- -\sum\nolimits_{i,l,r} c_{imrl}\lambda_{imrl} \geq FC\cdot t, \quad (7)$$

where $\pi_0 = B_0$.

Inequality (8) is intended to prevent a situation where the volume of raw materials in the warehouse exceeds the maximum capacity:

$$\sum\nolimits_l b_{l(m+t)} \leq \bar{b}. \quad (8)$$

Formulas (9–10) is a system in which the drawn values reflect the maximum travel time of the lot:

$$\tilde{T}_{r\tilde{m}} = m^*: \begin{cases} \left| L_r - \sum\nolimits_{\underline{m}=\tilde{m}}^{m^*} S_{\underline{m}} \right| \to \min \\ L_r - \sum\nolimits_{\underline{m}=\tilde{m}}^{m^*} S_{\underline{m}} \leq 0 \end{cases} \quad (9)$$

$$S_{(m+t-1)} \sim LN\left(\tilde{a}_{(m+t-1)}, \delta_{(m+t-1)}\right). \quad (10)$$

Formulas (11–12) specify an estimate of the coefficient of the useful volume of raw materials that will reach the warehouse. Formula (11) uses the function arctg(x) to reflect the useful volume of raw materials that has reached the warehouse. Since the function y = arctg(x) can take values in the interval [0; $\pi/2$], $x \in [0; \infty]$, it is necessary to make changes to the relationship between the value of the coefficient[4] of raw materials reaching the warehouse and how much time has passed since the moment the lot appears on the exchange[5] in such a way that the condition is met: $0 \leq \operatorname{arctg}(x) \leq 1$, $x \in [0; \infty]$, $\beta = \text{const}$, $\tilde{\beta} = \text{const}$:

$$\tilde{\gamma}_{\tilde{m}(m+t-1)} = \min\left(1; \max\left[0; 1-\frac{2}{\pi}\operatorname{arctg}\left(\tilde{\beta}\left(\left(m+t-1\right)- -\tilde{m}\right)\right)+\tilde{\varepsilon}^{(3)}\right]\right), \quad (11)$$

$$\tilde{\varepsilon}^{(3)} \sim U(\widetilde{left}), (\widetilde{right}). \quad (12)$$

Formulas (13–14) limit the values of some variables:

$$0 \leq x_{k(m+t-1)} \leq H_{k(m+t-1)}, \quad (13)$$

$$\varepsilon_{l(m+t-1)}^{(j)} \geq 0, j = 1{:}2. \quad (14)$$

After the solution, the following key parameters are calculated (15–18):

$$\pi_m = \pi_{m-1} + \sum\nolimits_k p_{km} x_{km} - \sum\nolimits_{i,l,r} c_{imrl}\lambda_{imrl} - FC, \quad (15)$$

$$\gamma_{\tilde{m}(m+t-1)} = \min\left(1; \max\left[0; 1-\frac{2}{\pi}\operatorname{arctg}\left(\beta\left(\left(m+t-1\right)- -\tilde{m}\right)\right)+\tilde{\varepsilon}^{(3)}\right]\right), \quad (16)$$

$$b_{lm} = b_{lm-1} - \sum\nolimits_k A_{lk} x_{km} + \gamma_{\tilde{m}m}\left(\sum\nolimits_{i,r} V_{i\tilde{m}rl}\lambda_{i\tilde{m}rl}\right), \quad (17)$$

where $\tilde{m} = m - \tilde{T}_{r\tilde{m}}$;

$$m = m + 1. \quad (18)$$

Recalculation of the main indicators occurs after the solution has been found for day m (15–17). In equality (18), the equal sign is used as an assignment operator to the value to the left of the equal sign to the value to the right.

4. Model calibration

To test the model, one of the leading forestry enterprises of the Primorsky Territory was chosen. Enterprises from four regions are most represented on the

[4] We will assume that this coefficient is in the range from 0 to 1.

[5] Let us introduce the assumption that the raw materials indicated in the lot were produced on the same day on which the lot was put up for auction.

exchange: Irkutsk Region ($r = 1$), Perm Region ($r = 2$), Republic of Buryatia ($r = 3$), Moscow Region $r = 4$). The planning horizon lies between February 1, 2022 and mid-May 2022.

Knowing the coordinates of enterprises, it would be possible to conduct a dialogue with them directly, bypassing the stock exchange. However, the exchange hides the real coordinates, so all transactions are carried out through the exchange, both from the buyer and from the supplier [11].

An array of the following data was collected from the official website of the exchange and from the enterprise for the specified period: prices of proposed applications c_{irm}, volumes of applications v_{ilrm}, selling prices of final goods p_{k1}, number of applications for each type of raw material. Since the exchange website runs software that does not allow data to be collected automatically, we can conclude that the data is not allowed to be used in large quantities, so all of the above values will be noisy and slightly changed.

The main initial data[6] characterizing the enterprise are presented in *Tables 1* and *2*.

For calculations, we will use the high-level programming language Matlab and the function from the intlinprog[7] extension package to find solutions to linear optimization problems.

Evaluating[8] function $\tilde{b}_m\left(\{V_{imrl}\}_{i\ r}, \{c_{imrl}\}_{i\ r}, m\right)$ with input parameters $\left\{\{V_{imrl}\}_{i,r}, \{c_{imrl}\}_{i,r}, m\right\}$, neural networks[9] (NN) with the following properties[10] were used: 10 hidden layers with activation function tg(x), 1 output layer (ReLU), learning algorithm – Levenberg–Marquardt. Coefficient of determination (Euclidean Metric) $R^2 = 0.78$.

Table 1.

Basic initial parameters of the enterprise

Parameters, units of measurement	Values
p_{km}, rub	$\forall m$: (1; 1,5; 1,6; 1,7) · 10^4
$N^{(j)}$,conventional units	$15p_{km}$
$\tilde{T}$, days	22
M, days	100
$\bar{b}$, m^3	7000
FC, rub	100 000
B_0, rub	3 000 000
L_r, km	(3740, 7560, 3250, 9000)
H_{nk}, units	$\forall m, k$: $H_{km} = 4$

Sources: enterprise, author.

Table 2.

Cost of raw materials per unit of production

		Item number (*k*)			
		1	2	3	4
Raw material type number (*l*)	1	2	3	4	3
	2	1	3	3	5

Sources: enterprise.

[6] The author does not have the right to publish some of the company's data due to his signing an agreement "On non-disclosure of trade secrets". In this regard, the author apologizes to students of the current work.

[7] https://www.mathworks.com/help/optim/ug/intlinprog.html

[8] Data for calculations were taken from the Enterprise and from the archive of exchange publications.

[9] https://www.mathworks.com/help/deeplearning/ref/fitnet.html

[10] The use of NN is since classical methods of regression analysis did not give positive results.

5. Discussion

Let's look at how the volume of raw materials in the warehouse changed (*Figs. 1, 2*). The behavior of raw materials in the warehouse is stable on average, in contrast to the results of searching for an optimal solution [13], where the maximum value of the total stock of raw materials is reached closer to the 70th day. Moreover, rare emissions show that the state of the levels of raw materials in the warehouse are stable, which allows the enterprise to optimize the management of the warehouse and allocate from it the required warehouse volume as precisely as possible and use the rest for other needs without the need for expansion or vice versa. *Figure 3* shows the production volumes of goods. It can be seen that on average production volumes tend to a maximum of four units. Rare emissions indicate that production is operating stably. When searching for an optimal solution, work [13] shows that production volumes are always equal to 4. The results of the current solution search method are quite close to the optimal one, in terms of production volumes.

The most important indicator remains the value of profit over the entire planning horizon. To estimate the profit trajectory, consider *Fig. 4*. We denote by $opt_m(e)$ the optimal profit value on day m for data sample e.

The behavior of the average values of profit volumes shows that the share of deviation from the optimal solution is not significant and amounts to 0.1964 at the end of the planning horizon. The profit value is growing steadily.

We consider the positive, negative aspects and directions for further development of the current model. The positive aspects of the model include:

1. The conceptual simplicity of the research in terms of modeling – methods for optimizing linear problems have received serious study in science.
2. Considers many important aspects of the forestry industry, for example, the useful volume of raw materials that will arrive at the warehouse, the time of the lot in transit.
3. Allows you to make decisions on each planning day, which corresponds to how the process of production planning and the formation of raw material supply chains occurs.
4. As an analysis of the resulting solution using the example of one of the Primorsky Territory enterprises and exchange data showed, the profit received does not differ much from the optimal value.
5. This model may include other processes of the forestry industry such as logistics, cutting problems and others.
6. As follows from [12, 13, 34], it is necessary to consider changes in railway capacity when forming supply chains. This work takes this feature into account (restrictions (9–10)).
7. The behavior of the values of raw materials in the warehouse is more predictable and stable than with the optimal one, but at the same time there is a payment in profit values.

The negative aspects of the study include the following points:

1. The nature of the β value when calculating the useful volume of raw materials that will reach the warehouse is not known with certainty. It may well be that β is not a parameter, but some function dependent on time or other factors.
2. The work does not consider an important factor in the process of calculating the cost of goods on each individual day or for a period over the entire planning horizon.
3. An important factor that can greatly affect profit – the logistics of finished goods to points of consumption – is not considered in the work, but it is indicated that the model can be modified to a state where it will include this subtask.
4. The key factor in the economy has always been demand. This work does not reflect the influence of demand on the production of goods.
5. Forestry companies often purchase raw materials in a B2B format. It would be useful to also use this tool to diversify the risk of raw material supply chains.
6. Often decisions may have different risk measures. This allows you to get more profit with a competent approach. It would be useful to modify the model

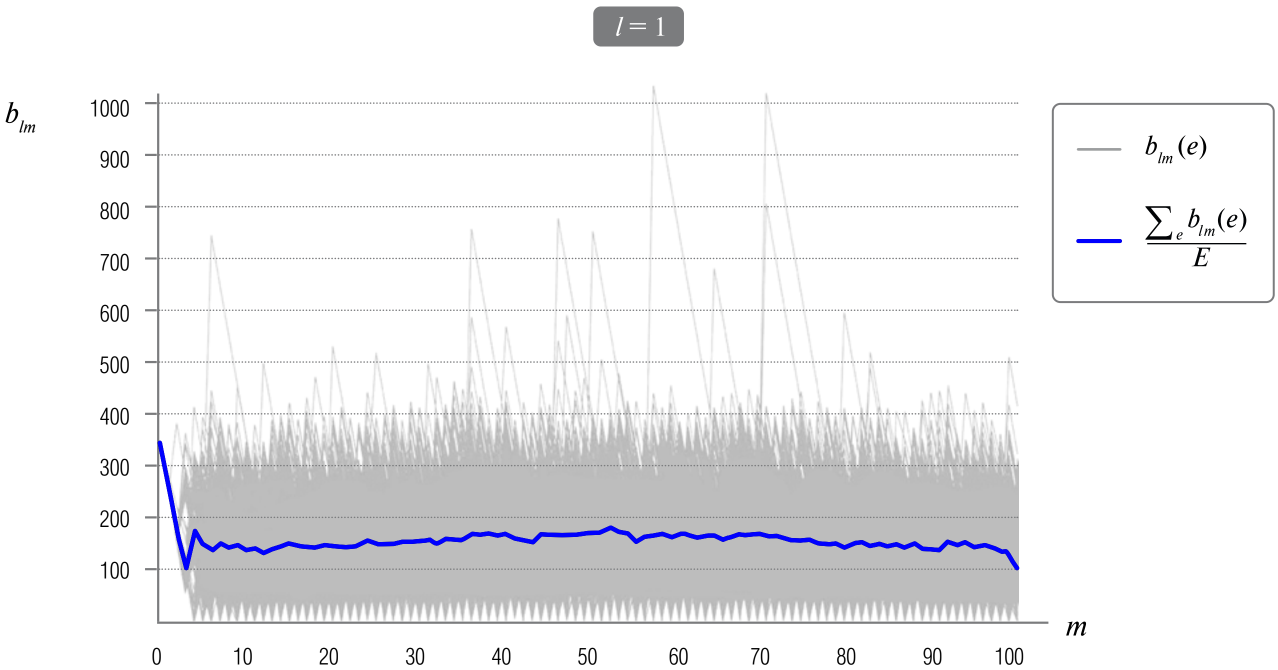

Fig. 1. Visualization of the behavior of the trajectory of raw material inventories of type $l = 1$ in a warehouse over the entire planning horizon.

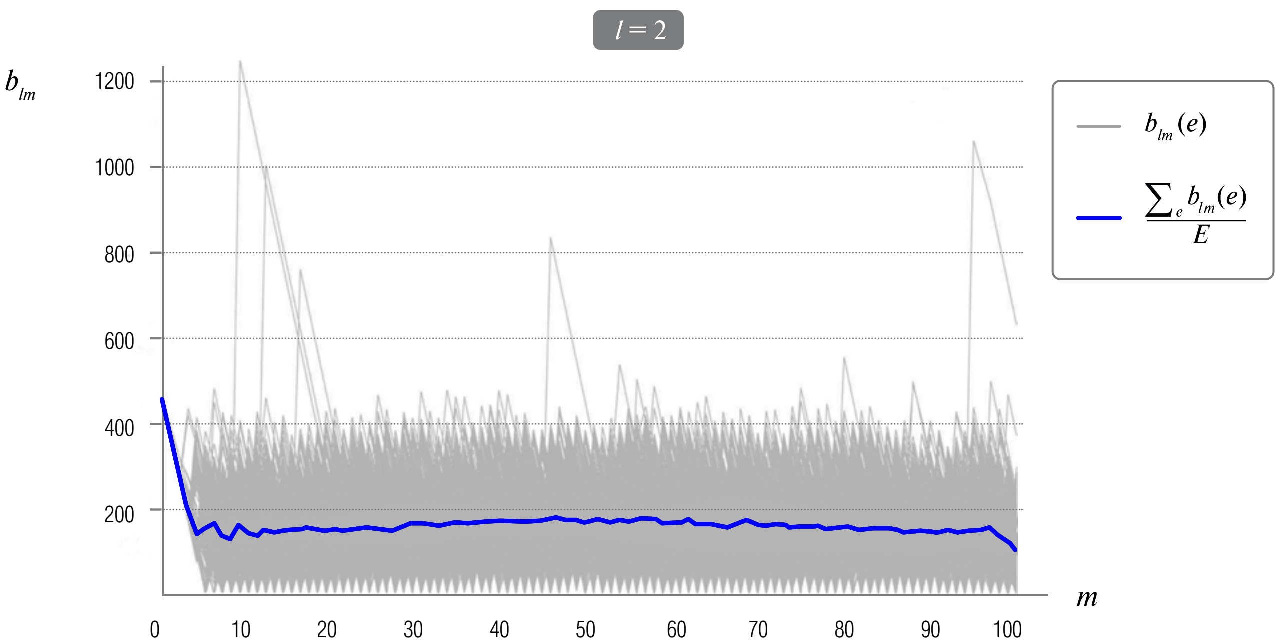

Fig. 2. Visualization of the behavior of the trajectory of raw material inventories of type $l = 2$ in a warehouse over the entire planning horizon.

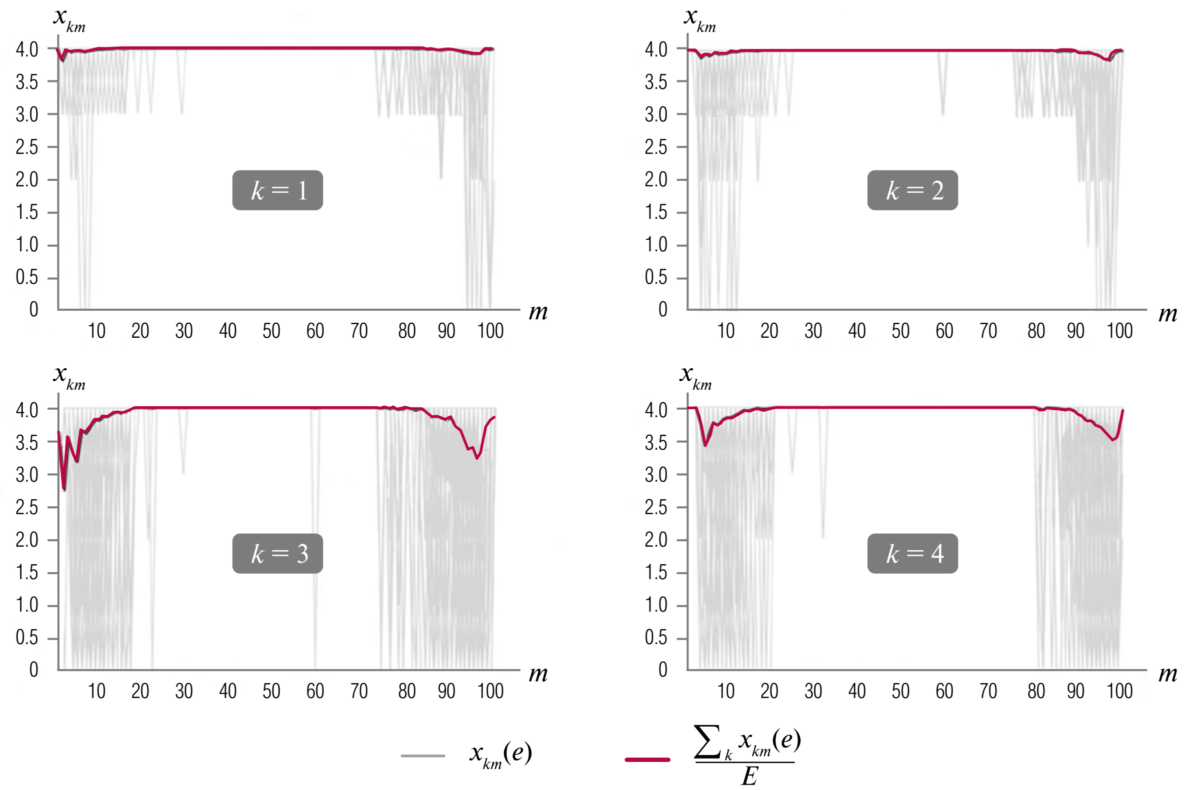

Fig. 3. Visualization of production volumes of goods for each type.

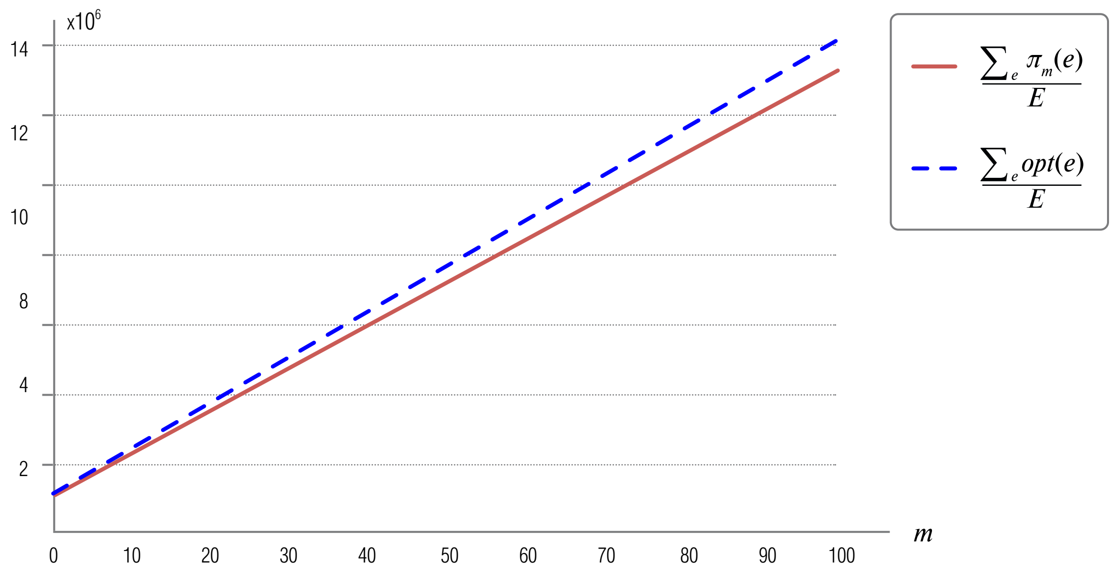

Fig. 4. Visualization of the behavior of the trajectory of average profit volumes over the entire planning horizon.

so that the decision maker has the opportunity, at a given level of risk, to form supply chains for raw materials.

7. The assessment of parameters ($\tilde{a}_m$, $\tilde{\delta}_m$) responsible for calculating the distance traveled by lots should be calculated not based on the experience of the enterprise (there is a high probability of making an error in planning), but using mathematical methods, for example, neural networks.
8. 2021 and 2022 have shown how important it is to be able to form and rebuild supply chains, including raw materials, under the influence of "black swans" (Black Swan Theory) and/or "rhinos" (Rhino Problem). The current work does not present any considerations on this matter.
9 It is not clear how the model will work if the quality of the estimate of the total volume of raw material stock in the warehouse ($\tilde{b}_m\left(\{V_{imrl}\}_{i\ r}, \{c_{imrl}\}_{i\ r}, m\right)$ and $\tau(m)$) will be low, and will the model show the same results in this situation.

Conclusion

This paper examines a model dedicated to the problem of optimal production management of a timber processing enterprise in matters of calculating production volumes and forming supply chains for raw materials under conditions of uncertainty. The model allows you to maximize the value of profit before tax and is a multi-period linear programming problem characterized by the ability to make decisions simultaneously on both issues under consideration: calculating production volumes and forming supply chains. The results of the model's implementation include the production structure and the sequence of purchasing lots from the commodity exchange, as well as the date of the last receipts at the warehouse. Multi-periodicity is achieved by dividing a task into many smaller ones to solve it on each individual day, just as it happens in enterprises.

The solution search process is a sequential process of solving linear programming problems for making decisions on production volumes and forming supply chains on a daily basis. Every day, the enterprise makes a decision based on estimates of when the goods will arrive at the warehouse and in what volume, where the latter decreases during the delivery process under the pressure of external mechanical factors. To achieve closeness of the solution to the optimal one, it was decided to calculate the regression of the dependence of the target total volume of raw materials in the warehouse on each individual day on the values of the current available lots and the day number. Also for planning, another regression was calculated, which allows us to estimate the dependence of the coefficient of change in the total volume of raw materials in the warehouse on the day number. All regressions were calculated on the data of optimal trajectories of raw material reserves, which were obtained by using one of the well-known models for searching for the optimal solution to the current problem in a complex manner over the entire planning horizon, which took all the necessary played values for the previous period.

The methodology developed for finding a solution was tested using the example of a timber industry enterprise in the Primorsky Territory. Based on the calculations carried out and the solution found, the company's recommendations for cooperation with the Russian Commodity and Raw Materials Exchange were formulated. Analysis of the decision showed that, despite the territorial proximity of the Irkutsk Region to the Primorsky Territory, it is worth paying attention to the purchase of raw materials from the Moscow Region and the Republic of Buryatia. This is explained by many reasons, among which the most important can be identified: sufficient potential in terms of extracted raw materials and a more acceptable pricing policy of companies. A brief analysis of the possible production volumes of each type of product was carried out. Analysis of the solution found demonstrates that the production of most types of goods should be maximum over the entire planning horizon, with rare exceptions.

The positive and negative aspects of the model are given, and ideas for its further development are considered.

In general, it can be argued that the model we developed is effective in finding a solution to the problem. ■

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